§1. Sight Reduction/Great Circle Route Calculations

The basic method for solving a spherical triangle was worked out in ancient Greece. Different variations of this solution have been formulated to try and optimize for different tools and scenarios.

The solution that gives the best results when doing calculations with a slide rule was developed in 1920 by Captain L.G. Bygrave[1] of the Royal Air Force. Plotting a great circle route is precisely the same calculation as you would use for sight reduction of a celestial sight.

On balance, the Bygrave equations will yield more accurate results when calculated on a slide rule than the more usual sine/cosine equations.

Given that this is a 10" rule, the full range of sine values = scale ST + scale S = 20 inches. The full range of tangent values = scale ST + scale T_2 + scale T_3 + scale T_4 = 40 inches.

The Bygrave equations still use cosine values, and like sines, all the solutions need to fit into 20" of rule length. But a person can dial in his tangent values more precisely, using all 40" of rule length. So at least *some* of the values you use will be more precisely set than if you are using sine/cosines only.

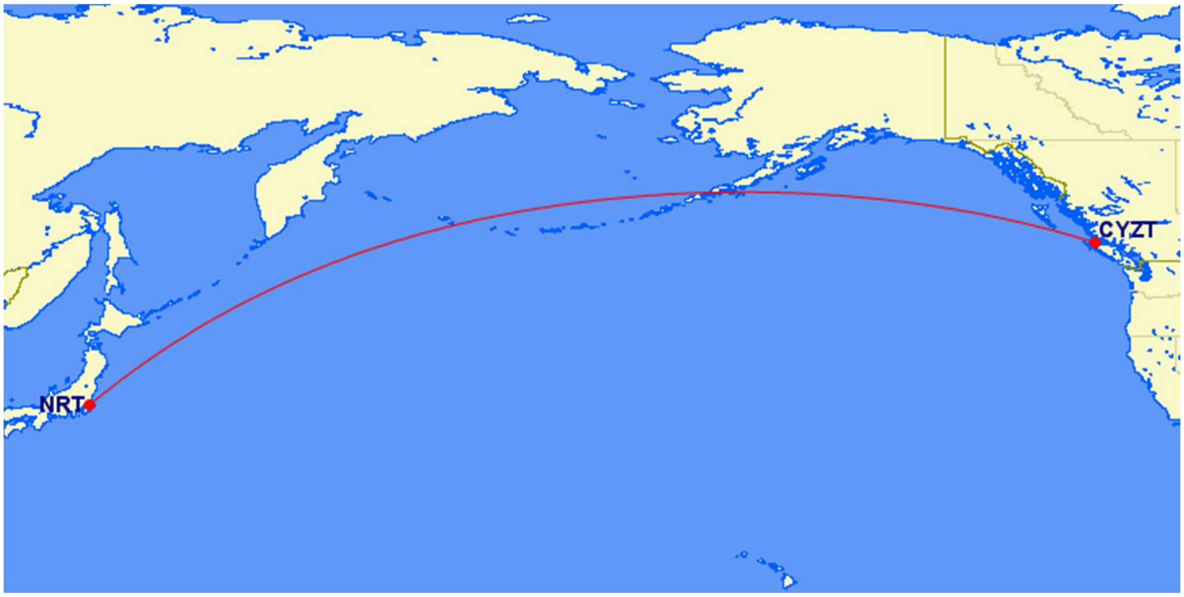
If you wish to sail from Port Hardy, on the northern tip of Vancouver Island, to Tokyo, your "straight line" course is called a great circle route.

---

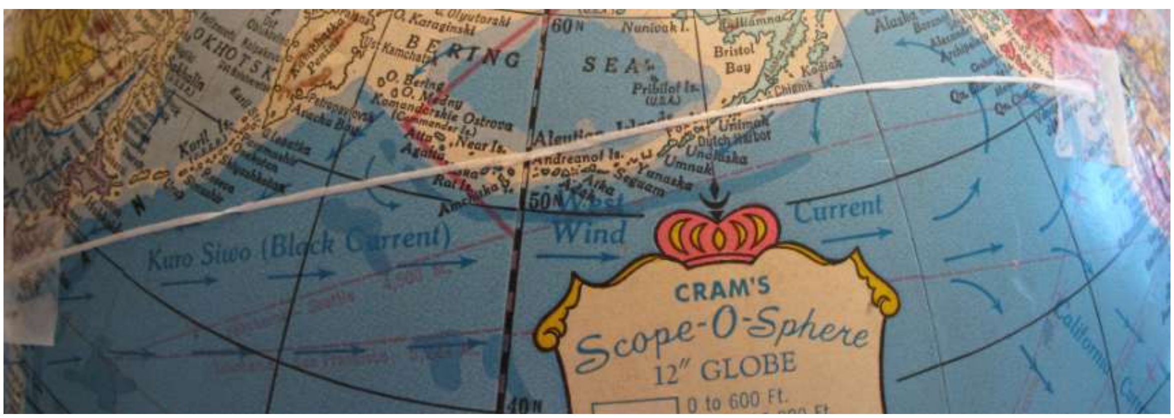
[1] Captain Bygrave's original slide rule was actually cylindrical rather than straight. The advantage of this design was that the scales, if unwound, would be many feet long. It gave much better accuracy than a 10" straight slide rule. You can see a picture of one at https://en.wikipedia.org/wiki/Bygrave_slide_rule.

Gary LaPook has done extensive research and writing about the Bygrave, and the worksheet in this section for solving the navigational triangle, is entirely derived from Gary's website at http://tinyurl.com/BygraveSlideRule.

I am deeply indebted to Gary for coaching me on how to do great circles/sight reduction with the Bygrave equations.

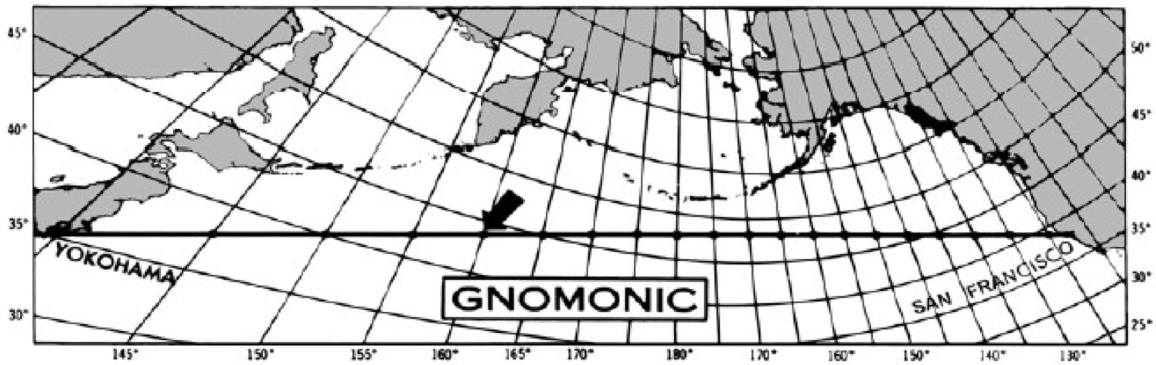


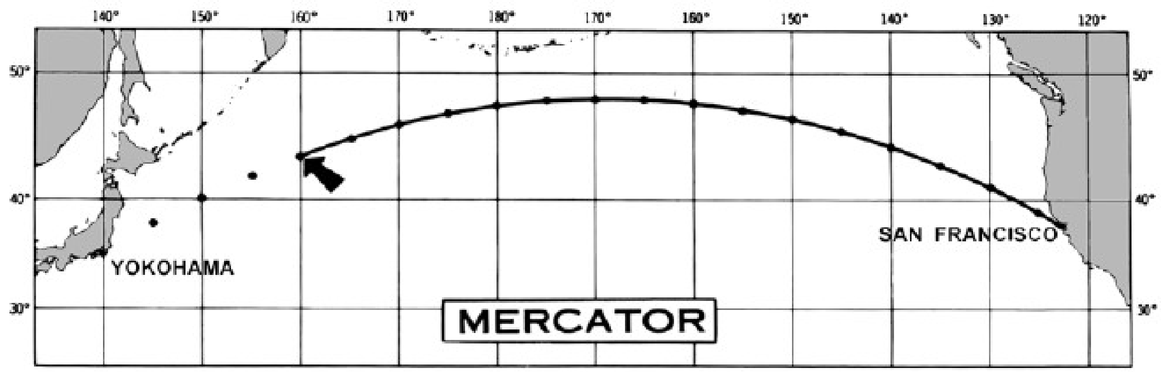
Such a course looks like a curve on a Mercator chart, as above. But if you get a globe and tape a piece of dental floss from your starting port to your destination, you can visualize that the shortest distance from Port Hardy to Tokyo will indeed take you through the Bearing Sea.



Figuring out what course to steer to be most efficient can be a bit of a challenge, insofar as the compass course is slowly but continuously changing.

You can purchase a gnomonic chart, of course.





But frankly, I have never much cared for gnomonic charts. They are a cool idea, but costly, and big enough to be awkward to use in a 45 foot yacht. Further, depending on your cruise plan, you may need several of them. Big container ships can pick a course and stick with it across an ocean, but cruising sailors tend to be always working compromises between wind/weather/shortest-course. Depending on the location of the North Pacific High, a sailing yacht on its way from Hawaii to Vancouver may go almost as far north as the Aleutian Islands before picking up the westerly breezes that let it head for home. This can lead to an untidy gnomonic chart as you keep replotting great circle courses from locations you never expected to be at.

On the other hand, I can do a great circle calculation in 4 minutes on a slide rule,[2] once a day, and recalculate an almost[3] perfect great circle route, from anywhere to anywhere.

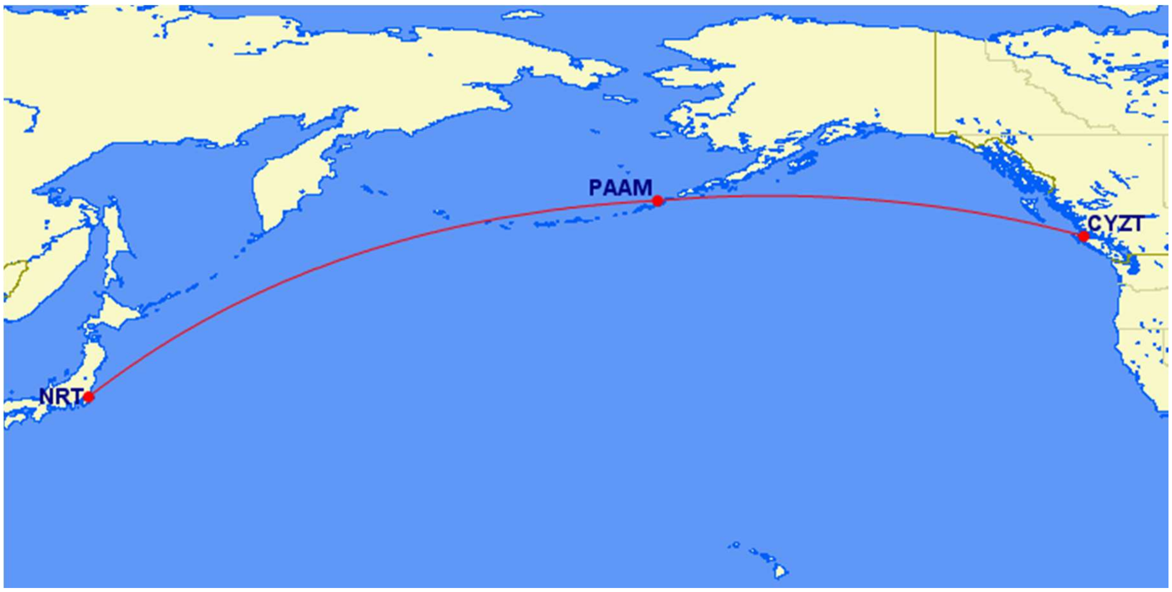
---

[2] I can also do it with an electronic calculator, of course. But there is no time savings for me. It takes 4 minutes to work out on a calculator, and 4 minutes to work out with a slide rule. It is more *fun* to use the slide rule.

[3] The earth is big enough that a rhumb line course is virtually identical to a great circle course for distances less than 200 nm. So a once-a-day recalculation of the course to sail will work out just fine. Frankly, those who use gnomonic charts often only calculate their course changes for once every 5° of longitude. The man with a slide rule in his hand will steer smaller.

Besides, as will be seen below, once you figure out how to calculate a great circle route, you will automatically be able to do sight reduction of any star you wish without having to pack all six volumes (at 12.6 pounds!) of Pub. 229 along.[4]

Now, as I am planning my trip from Port Hardy to Tokyo, I discover that Dutch Harbor is almost directly on the great circle route I want to follow, so I decide that I will lay over there for a day or two to warm up, wash some clothes, then get a hamburger and fries after weeks of boat food.



Port of Departure:	Port Hardy, N 50° 43.3', W 127° 29.6'
Destination:	Dutch Harbor, N 53° 53.4', W 166° 32.5'

Inputs for calculation: the latitudes of both ports, plus the difference between the longitudes.

```
              166° 32.5'
             -127° 29.6'
Difference     39° 02.9' West (i.e. from Departure to Dest is west)
```

This difference is known as the "meridian angle" and is abbreviated as "H".

When subtracting a value of, for example, 23° 58' from 90°, it is easier if you first convert that 90° 00' to 89° 60'…an equivalent value, but easier to work with.

---

https://www.starpath.com/catalog/books/1898.htm

Because of the ease of use, even a slide rule lover like me will still want to keep at least volumes 2 and 3 of Pub. 249 aboard to use with sun/planets plus a number of the bright stars in the sky.

But for those occasional sights of stars that lie outside the range of declinations that Pub. 249 covers, your Mark 1 Navigator's Slide Rule will serve just fine.

$$\begin{array}{r} 90^\circ\ 00' \\ -23^\circ\ 58' \\ \hline \text{Hard to do} \end{array} \qquad\qquad \begin{array}{r} 89^\circ\ 60' \\ -23^\circ\ 58' \\ \hline 66^\circ\ 02' \end{array}$$

Port Hardy (N 50° 43.3′, W 127° 29.6′) to Dutch Harbor (N 53° 53.4′, W 166° 32.5′)

Great Circle Calculations/ Sight Reduction

If using for celestial SR, "Destination" = the GP of the celestial object.
Latitude Destination = Declination of object.
Longitude Destination = GHA.

My Longitude = W (E/W) 127° 29.6′ °
Longitude Destination = W (E/W) 166° 32.5′ °
Meridian Angle = W (E/W) 39° 03′ °

DATA INPUTS: Meridian Angle < 90°	Differences Where Meridian Angle > 90°
Meridian Angle (t) = W (E/W) 39° 03′ °	180°- Meridian Angle = (t) = ___ (E/W) ______ °
My Latitude (L) = N (N/S) 50° 43′ °	
Latitude Destination (d) = N (N/S) 53° 53′ °	

1. tan(d) ÷ cos(t) = tan(W)

   W = 60° 30′ °

2. Use [+W] if d has same sign as L.
   Use [-W] if d has opposite sign as L.

   (90° - L) ± W = X

   X = 99° 47′ °

3. Ignore the sign of X (i.e. -60 = 60).
   If X < 90°, then X = Y
   If X > 90°, then 180 - X = Y

   Y = 80° 13′ °

4. [cos(W) ÷ cos(Y)] * tan(t) = tan(Z)

   Z = 66° 53′ °

5. cos(Z) * tan(Y) = tan(Hc)

   Hc = 66° 17′ °

   Zn = 293° 07′ °

   Convert Hc from decimal deg to deg/min if plotting a celestial position.

   Hc = ____ ° ____ ′

6. (90° - Hc) * 60 = Distance in nm

   Distance = 1,422 nm

2. (… = X

= ______ °

89° 60′
-50° 43′
+60° 30′
99° 47′

Azimuth Rules for Step 5

Meridian angle (t)	1° to 179° W	1° to 179° E
L is in North Latitude		
If d or W > L	Zn = 360 - Z	Zn = Z
if d contrary or W < L	Zn = 180 + Z	Zn = 180 - Z
L is in South Latitude		
If d or W > L	Zn = 180 + Z	Zn = 180 - Z
if d contrary or W < L	Zn = 180 + Z	Zn = 180 - Z

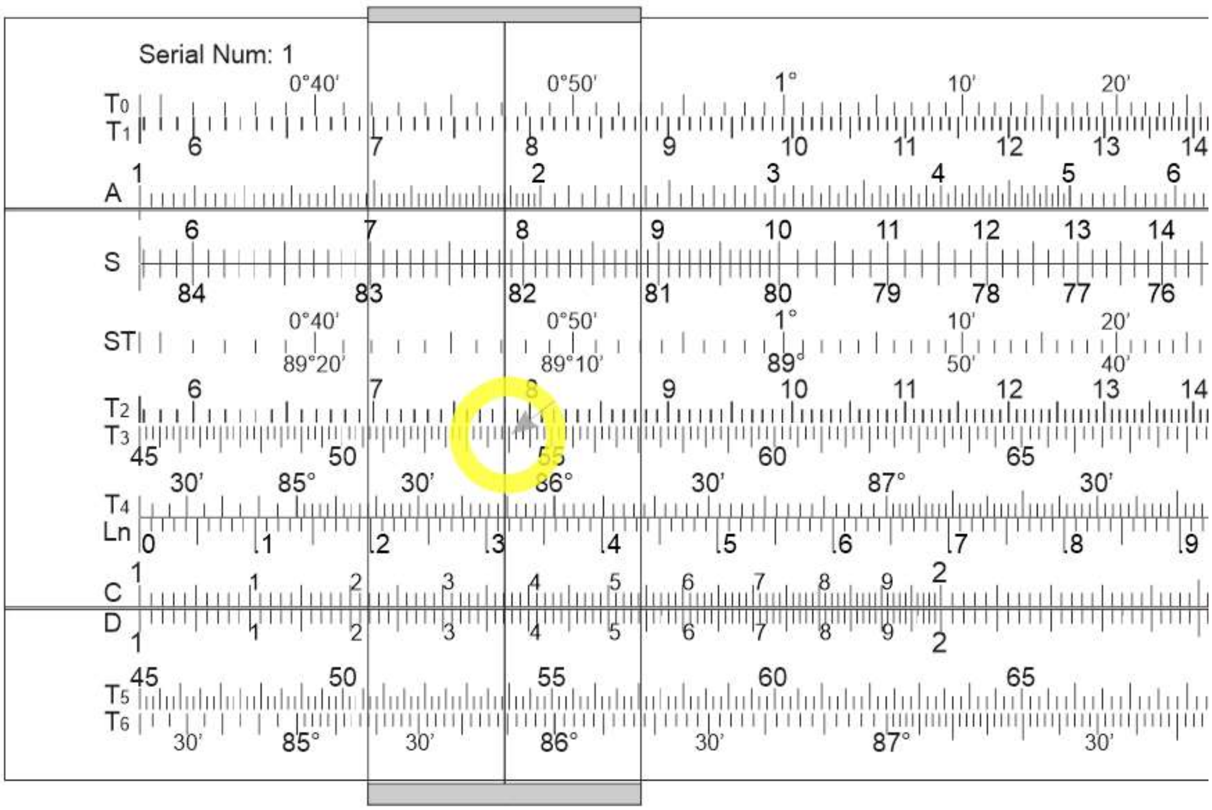
Equation 5 works for sight reduction of all sextant sights, as well as for great circle route calculations. Equation 6 works where the distance to be travelled is no more than 1/4 the circumference of the earth away from starting point.

Write down the data inputs: local hour angle,[5] meridian angle,[6] your latitude, and the latitude of your destination. You will use the left hand side of the form exclusively, since your meridian angle is less than 90°.

Step 1: tan(53° 53') ÷ cos(39° 03) = tan (W)

Center the slide in the body, and position the cursor over 53° 53' on the T3 scale.

There is no way, with slide rule accuracy, you can do anything about that final "point four" minutes of Dutch Harbor's latitude. Just come as close as you can.



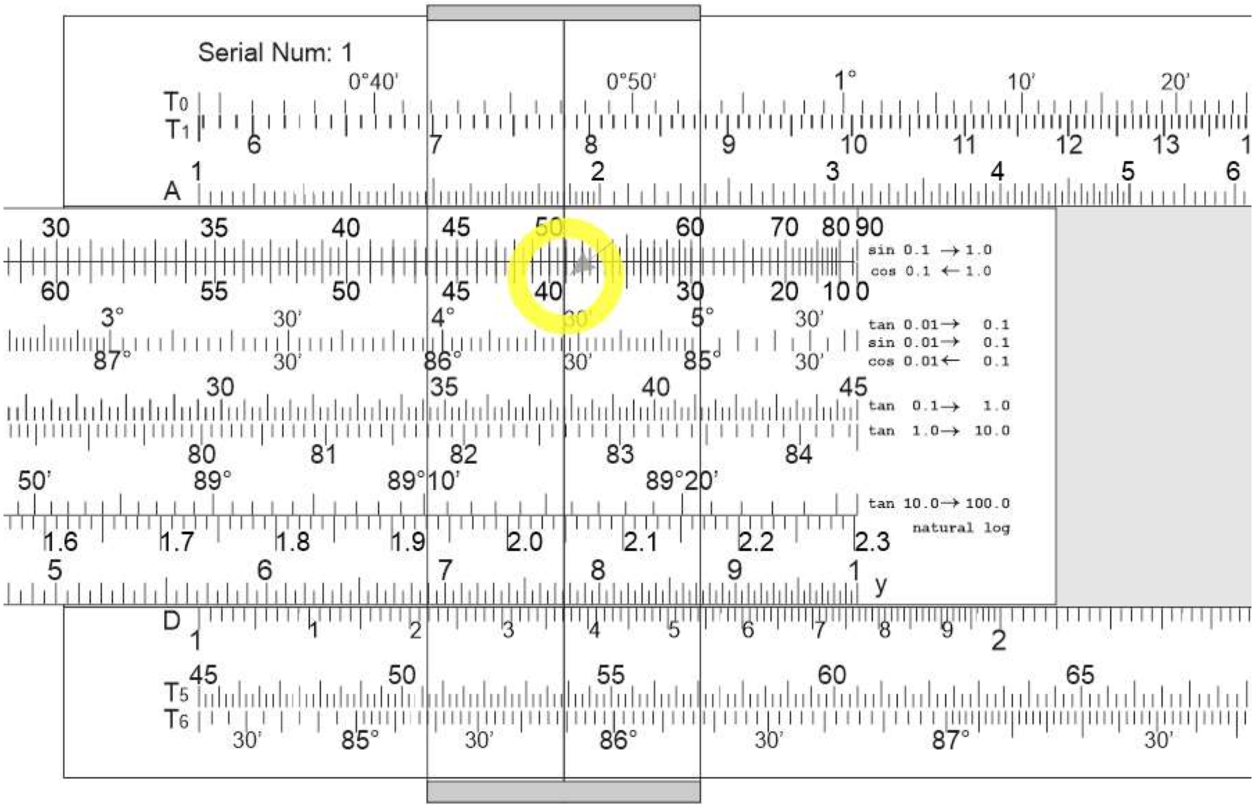
---

[5] LHA is the difference between your longitude and that of your destination, measured from your location in a westerly direction. So LHA 10° = a meridian angle of 10° W. LHA 350° = a meridian angle of 10° E.

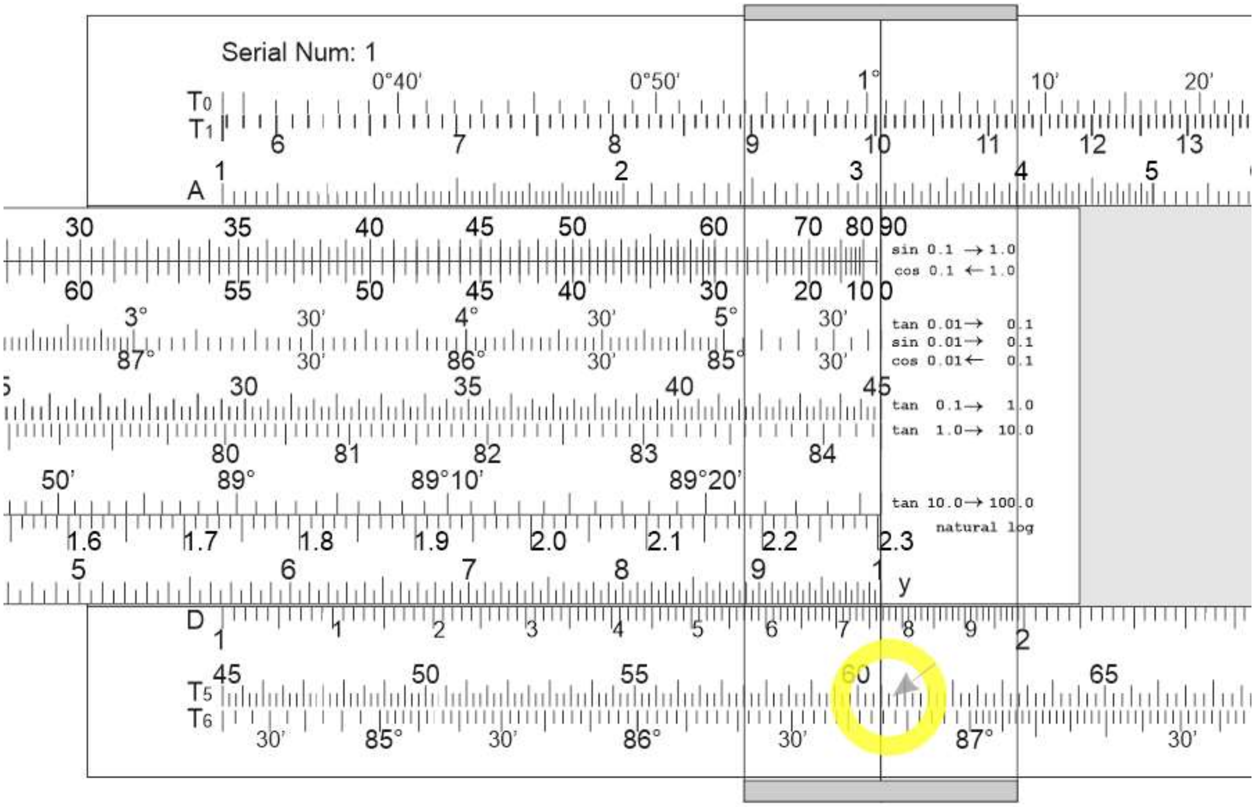
[6] If you are fuzzy on "meridian angle", it is the difference between your longitude and that of your destination, expressed as a value from 0° to 180°, going either east or west. Your location, your destination, and the north pole are the three points of the navigational triangle you are trying to solve.

If your entire training in celestial navigation involved the LHA from 0° to 360°, measured westward, then you have to reorganize your thinking a bit to use either a slide rule or a calculator to solve the triangle.

Position cos(39° 03') from the S scale underneath the cursor, in order to divide.



Move the cursor to the right index, and read 60° 29' on the T_5 scale.

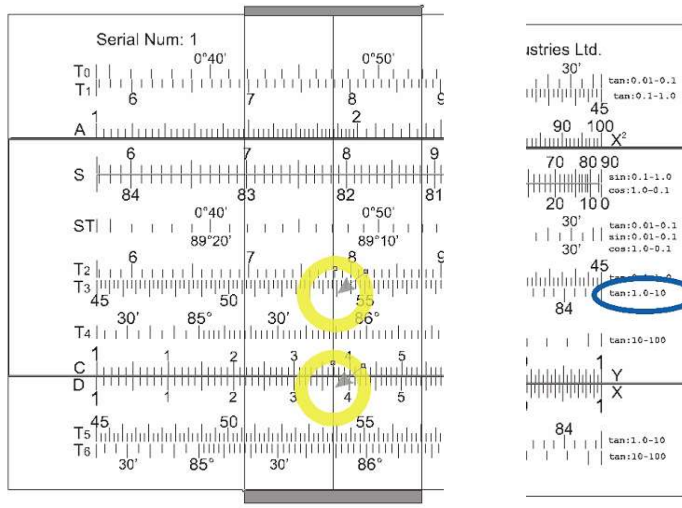


Which of the T-scales should I use to read my answer?

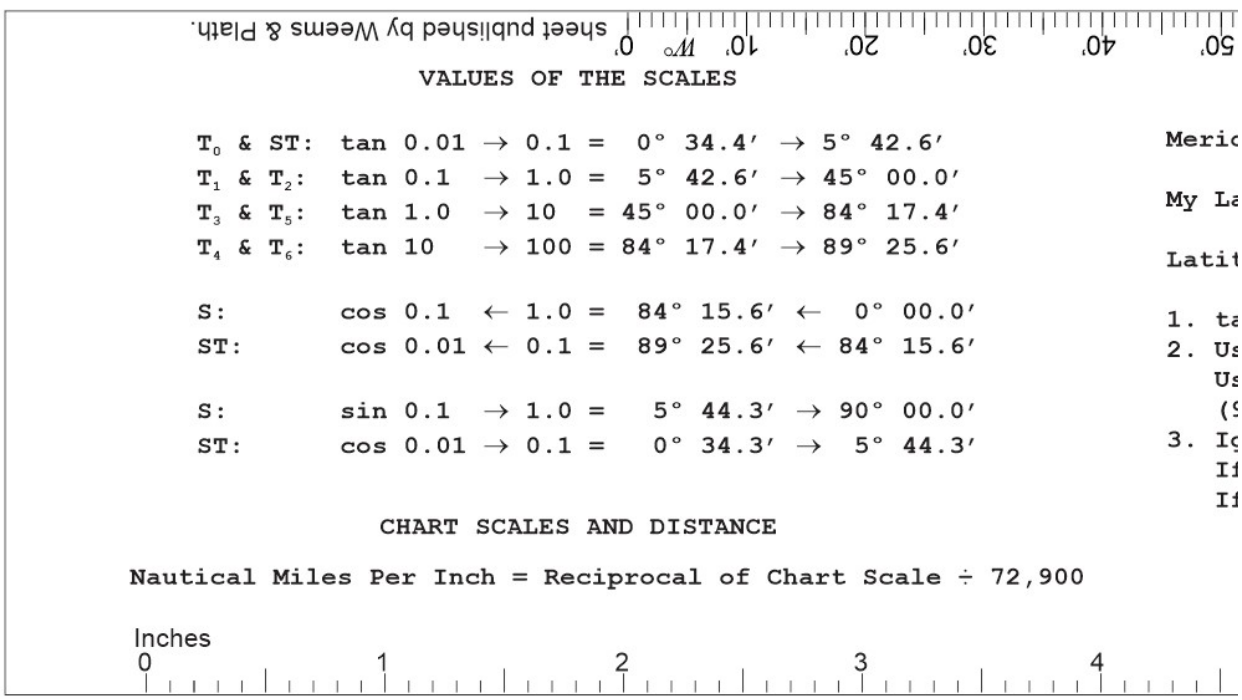
First, if you are doing these calculations once a day while sailing, the values will be in the same general range from one day to the next. Having calculated a great circle route on one day, you already know roughly what to expect when you recalculate the next day.

If you are starting this particular problem cold, however, you can get a sense of the range of your final answer by observing the numeric values of tan(53° 53') and cos(39° 03).

Note the value of tan(53° 53'), and also look at the right end of the T_3 scale.

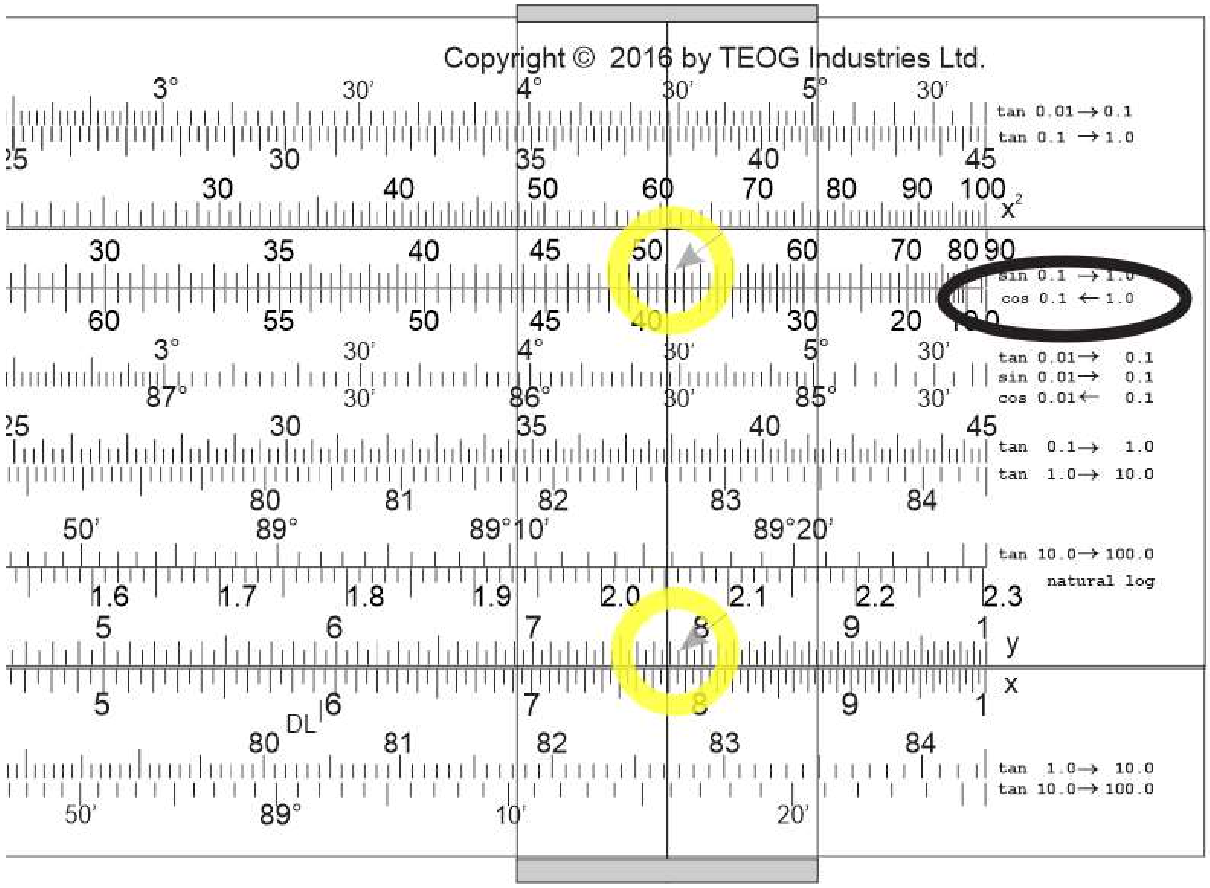


You can also flip the rule over.



You see that scale T_3 deals in values of 1.0 to 10. So the tangent of this angle is 1.37.

Look at the value of cos(39° 03'), and observe the cosine values on the right edge of the scale. Scale S deals in cosine values of 1 to 0.1.



So the value of cos(39° 03') is 0.776. This means that the equation of step 1 may be expressed as: 1.37 ÷ 0.776. I can know at a glance that .776 is going to go into 1.37 something in the neighborhood of 2 times.

Since scale T_5 deals in values from 1 to 10, I know that my answer of 2-ish needs to be read on scale T_5.

Once you get a feel for the kinds of answers that are in-range, you can multiply and divide cosines and tangents without concern for the values on the C or D scales.

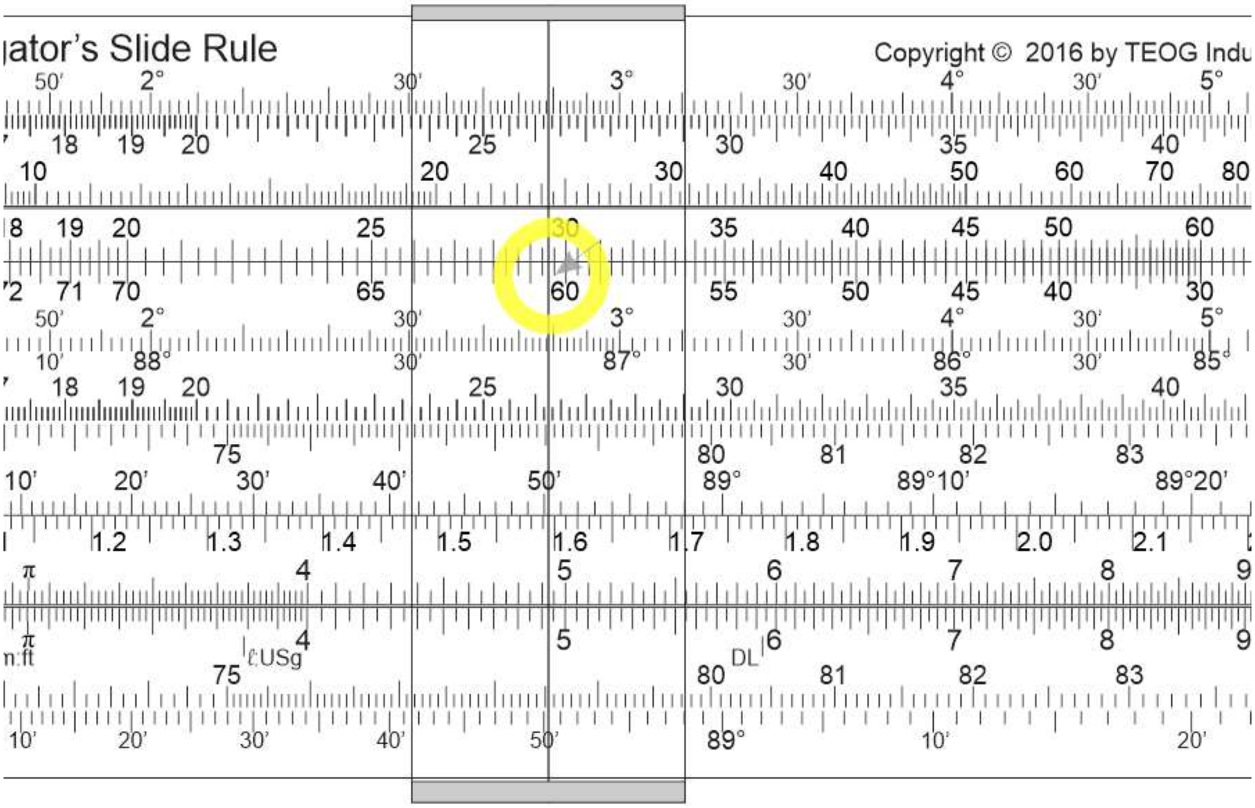
Section ***Error! Reference source not found.*** reviews the technique for doing mental math with trig functions.

Step 2: (89° 60' - 50° 43') + 60° 29' = 99° 46'

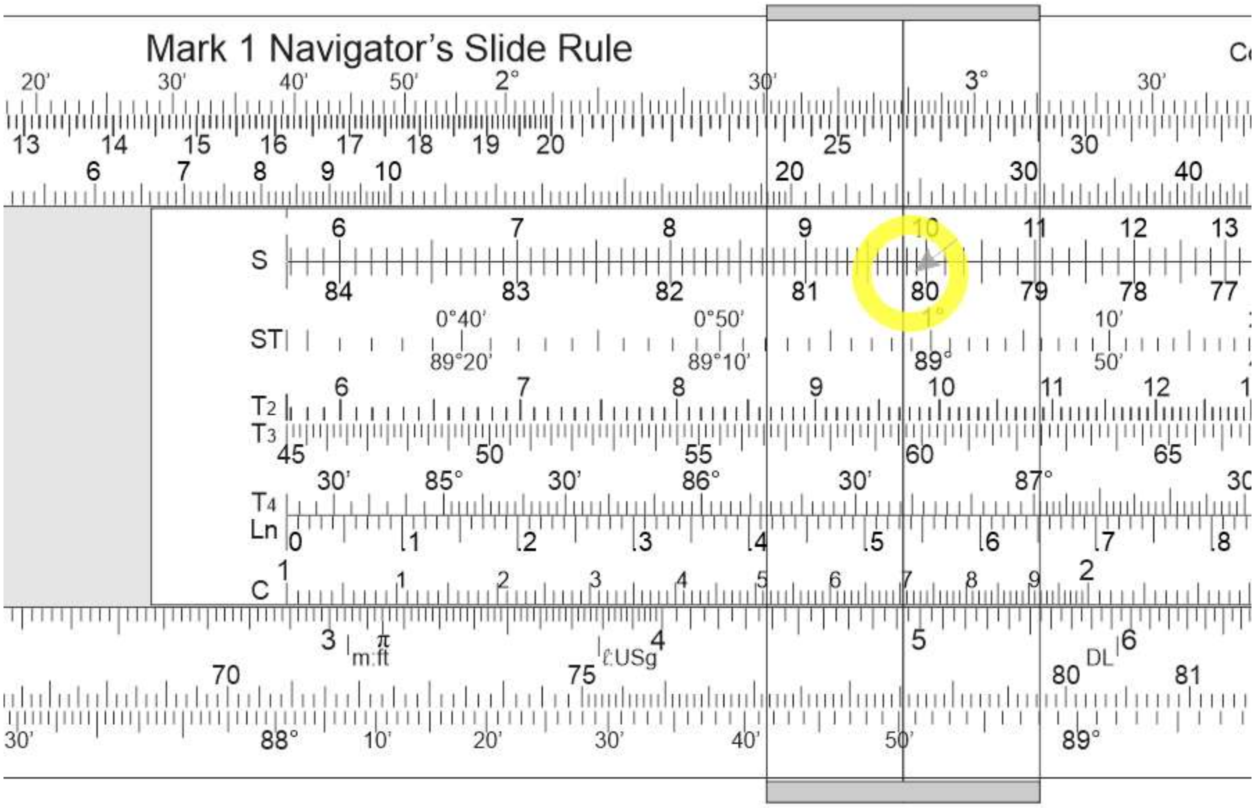
Step 3: 179° 60' – 99° 47' = 80° 13'

Step 4: [cos(60° 30') ÷ cos(80° 13')] * tan(39° 03') = tan(Z)

Center the slide, then position the cursor over cos(60° 30') on the S scale.

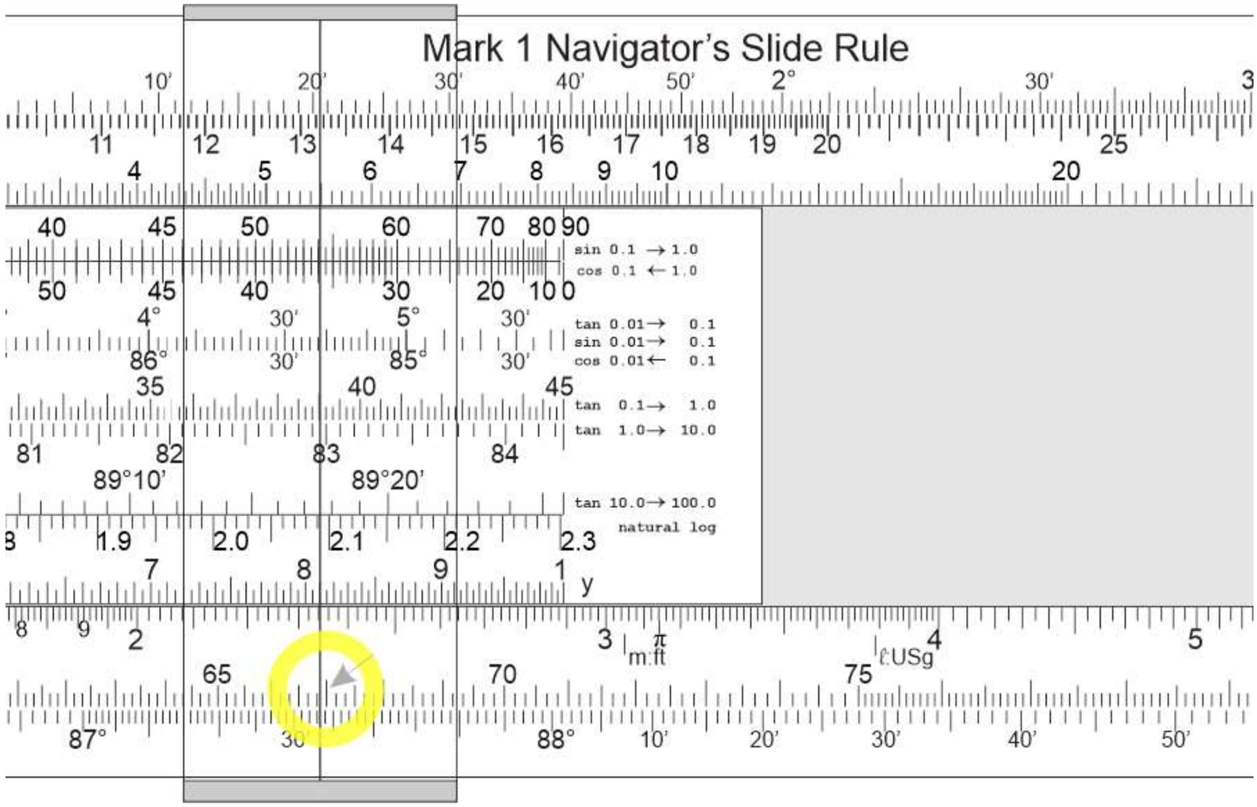


Move cos(80° 13') on the S scale beneath the cursor. This performs your division.



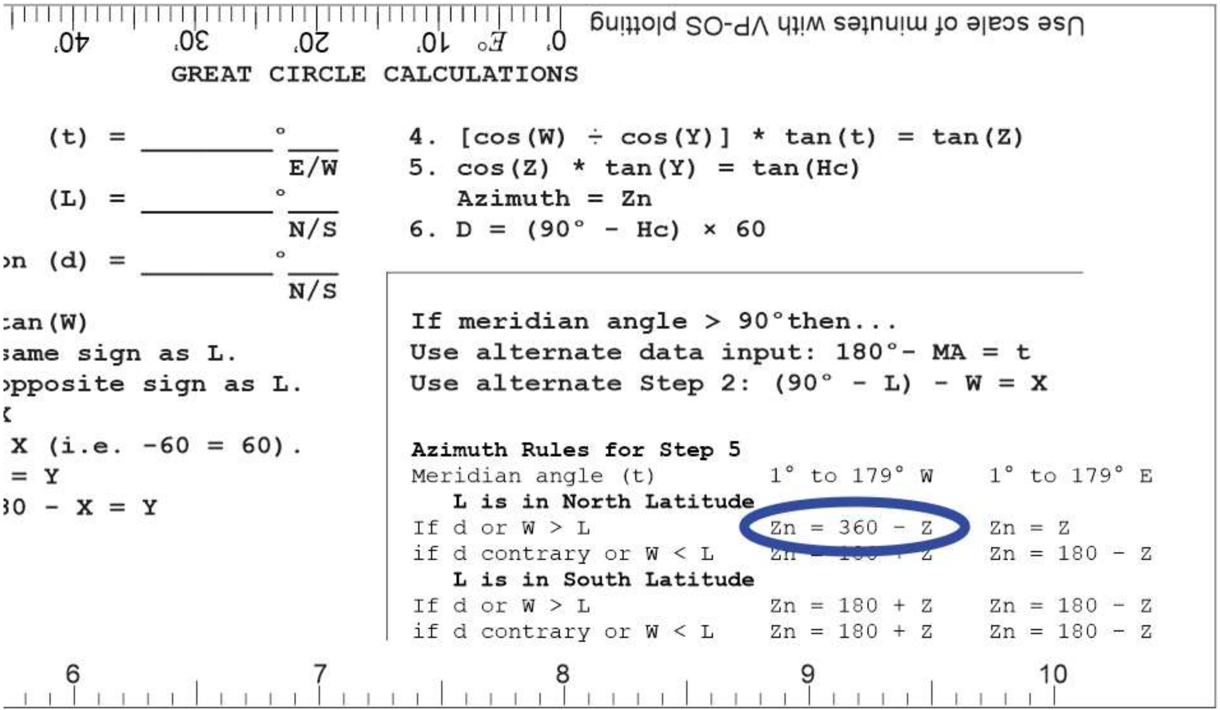
You are ready to go straight to multiplying now, so move the cursor over the top of tan(39° 03') on the T_2 scale...except you can't quite do that now. Tan(39) extends out too far to the right. So move the cursor to the left index, then move the right index on the slide to the cursor, and THEN multiply by tan(39° 03') from the T_3 scale, and read your answer on the T_5 scale:

Z = 66° 53'

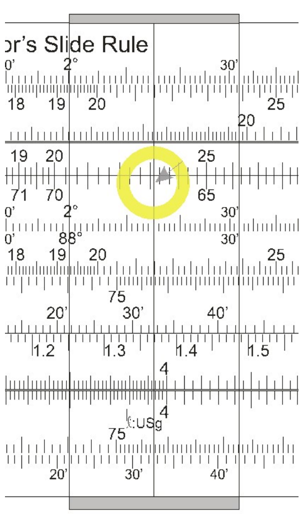


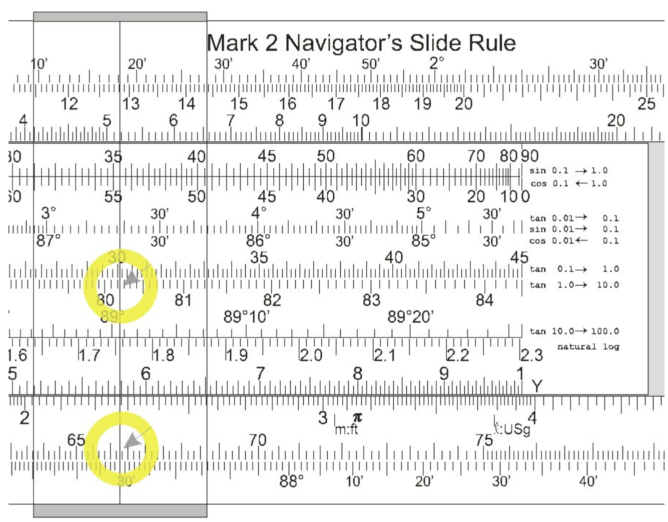
Convert Z to Zn using the chart on the worksheet or on the back of the rule. In this case, L is north of the equator, and d is greater than L (for that matter, W is also greater than L), and the LHA is 39°. So Zn = 360° - 66° 53'.

Remember that your math will work out more easily if you think of this as
359° 60' – 66° 53'.



Step 5: Center your slide, and position the cursor over cos(66° 53') on the S scale. Then move the right index on the slide under the cursor, and multiply by moving your cursor over tan (80° 13')



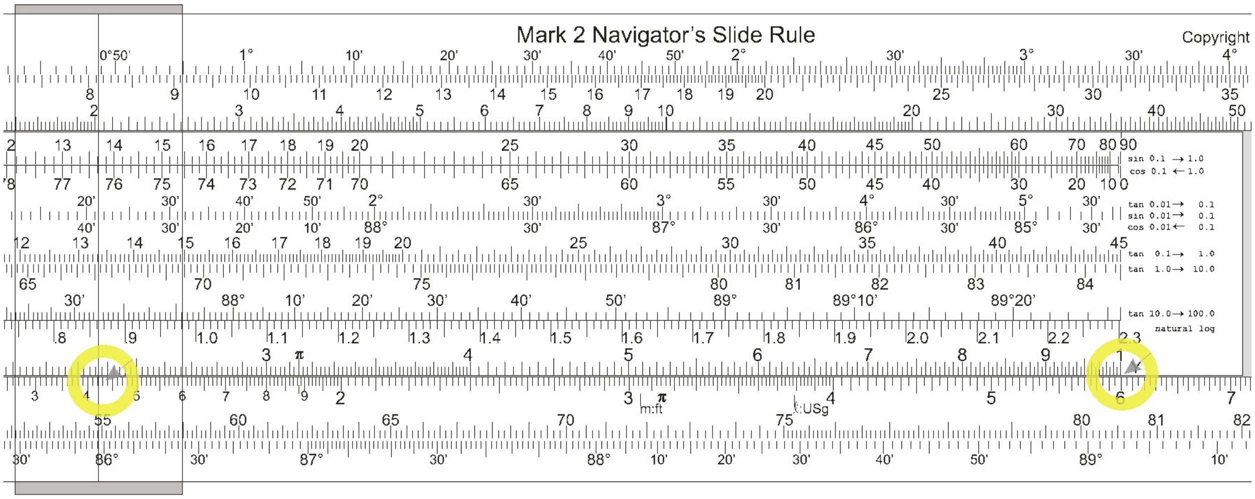


Hc = 66° 17'

Zn = 292° 42'

Step 6: 89° 60' – 66° 17' = 23° 43'. Convert this to decimal degrees = 23.7°

Multiply by 60, and get 1,422 nm.


Mark 2 Navigator's Slide Rule
Copyright
sin 0.1 → 1.0
cos 0.1 ← 1.0
tan 0.01 → 0.1
sin 0.01 → 0.1
cos 0.01 ← 0.1
tan 0.1 → 1.0
tan 1.0 → 10.0
tan 10.0 → 100.0
natural log
m:ft
l:USg