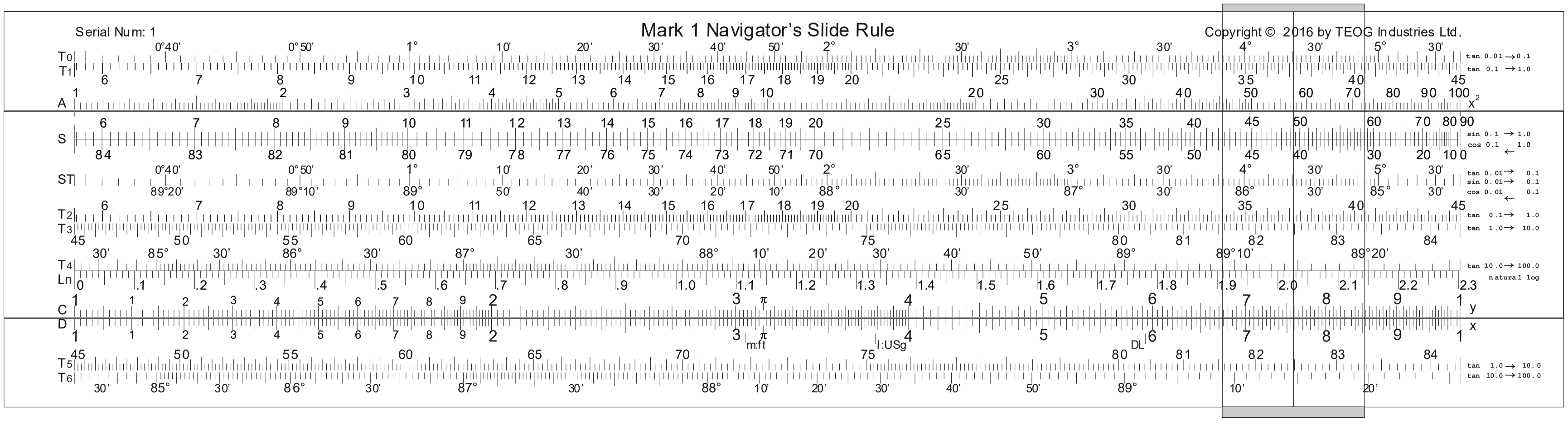
How to Use the *Mark 1 Navigator's Slide Rule*

Bob Goethe



Use scale of minutes with VP-OS plotting sheet published by Weems & Plath.

Values of the Scales

T_0 & ST:	tan 0.01 → 0.1 =	0° 34.4′ → 5° 42.6′
T_1 & T_2:	tan 0.1 → 1.0 =	5° 42.6′ → 45° 00.0′
T_3 & T_5:	tan 1.0 → 10 =	45° 00.0′ → 84° 17.4′
T_4 & T_6:	tan 10 → 100 =	84° 17.4′ → 89° 25.6′
S:	cos 0.1 ← 1.0 =	84° 15.6′ ← 0° 00.0′
ST:	cos 0.01 ← 0.1 =	89° 25.6′ ← 84° 15.6′
S:	sin 0.1 → 1.0 =	5° 44.3′ → 90° 00.0′
ST:	cos 0.01 → 0.1 =	0° 34.3′ → 5° 44.3′

CHART SCALES AND DISTANCE

Nautical Miles Per Inch = Reciprocal of Chart Scale ÷ 72,900

Inches 0 1 2 3 4 5 6 7 8 9 10

GREAT CIRCLE CALCULATIONS

Meridian Angle (t) = ________° E/W

My Latitude (L) = ________° N/S

Latitude of destination (d) = ________° N/S

1. tan(d) ÷ cos(t) = tan(W)
2. Use [+W] if d has same sign as L.
   Use [-W] if d has opposite sign as L.
   (90° - L) ± W = X
3. Ignore the sign of X (i.e. -60 = 60).
   If X < 90°, then X = Y
   If X > 90°, then 180 - X = Y
4. [cos(W) ÷ cos(Y)] * tan(t) = tan(Z)
5. cos(Z) * tan(Y) = tan(Hc)
   Azimuth = Zn
6. D = (90° - Hc) × 60

If meridian angle > 90° then...
Use alternate data input: 180°- MA = t
Use alternate Step 2: (90° - L) - W = X

Azimuth Rules for Step 5

Meridian angle (t)	1° to 179° W	1° to 179° E
L is in North Latitude		
If d or W > L	Zn = 360 - Z	Zn = Z
if d contrary or W < L	Zn = 180 + Z	Zn = 180 - Z
L is in South Latitude		
If d or W > L	Zn = 180 + Z	Zn = 180 - Z
if d contrary or W < L	Zn = 180 + Z	Zn = 180 - Z

Actual Size = 11.25” x 2.85”

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Preface

As you read these words, you may well be asking yourself, "*Why on earth would one use a* slide rule *to do navigational calculations, when you can get a* Casio fx260 Solar II *scientific calculator for just $10?*"

Not only is the fx260 inexpensive, it is seaworthy. One navigator experimented[1] and found that he could still work calculations on it, even while it was submerged in an inch of sea water.

To explain my motivation, I need to talk about myself a little bit. Bill Gates and I are only two years apart in age, but the similarities stop there. Bill built Microsoft, a company everybody has heard of. I built TEOG, a company nobody has heard of. In the 1980s, when WordPerfect was the king of the hill in productivity software, Bill began selling MS Word…a program that was far ahead of its time. I have brought out the Mark 1 slide rule forty-plus years after the electronic calculator wiped out the entire slide rule market.[2]

It is this difference in entrepreneurial acumen that goes a long way in explaining why Bill can write a personal check for $30 billion, while I am happy I can afford to order in pizza on Monday nights.

Creating a specialty slide rule is like painting landscapes in oil. The person who does so paints not because it will make him any money.[3] And his paintings are never as quickly or accurately rendered as a single snapshot taken with his iPhone.

No…a person paints landscapes because there is something in it that nourishes his inner man. At the bottom, there is an esthetic appeal to mathematics, engineering, and navigation that is every bit as real as the esthetic appeal of painting a landscape or creating a sculpture in marble.

I prefer to do my navigation calculations with a slide rule simply because it is *fun*.[4] And to take anachronistic practices a step further, I actually use a slide rule along with a *sextant*[5] to fix my position.[6]

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[1] fer3.com/arc/m2.aspx/Calculator-dunking-FrankReed-jun-2015-g31753

[2] Keuffel and Esser Co. presented the final slide rule they manufactured to the Smithsonian Institute in 1976. The Mark 1 you hold in your hands is one of only a handful of slide rules manufactured over the last 40 years.

The learning curve for using a slide rule was always steep, and it could manage only 3 or 4 significant digits in its answer. The electronic calculator was superior in so many ways.

You may have already realized, however, that while calculators are accurate to at least 8 significant digits, the last 5 of those digits often make little or no practical difference in navigation. The Empire State Building, the Golden Gate Bridge, and the Saturn 5 rocket that put men on the moon were all built with just 3 significant digits.

[3] Quite the contrary. If he really wanted to make money with a brush in hand, his best bet would be to offer to redo the siding of his local tattoo parlor.

[4] Playing with a slide rule on Sunday afternoon is about as practical/useful as watching football, which I also enjoy. We do the one for the same reason we do the other – for fun.

[5] Paul Hirose did a computer simulation of the Bygrave sight reduction formulas – which are the formulas used in this manual; see §44 – on a conventional 10 inch slide rule, and reported on his results here. Based on a run of 500,000 random sight reduction problems, he found that a slide rule delivered altitude (Hc) accuracy of 1.8', and azimuth (Zn) accuracy of 2.0'.

To approach this another way, the easiest and cheapest way to get nutritious, good tasting vegetables is to whip down to the local Safeway store, and pick some up.

But for all the supermarkets there are in the world, there are still some people who really enjoy growing a backyard garden. They take enormous satisfaction in eating their own tomatoes, and often will tell you that their tomatoes *taste better* than anything they can get in the store. They feel growing their own helps them get in touch with the earth, and may even help them center themselves spiritually.

In my case, working with slide rule and sextant gratifies both my curiosity and my soul, as I find myself in touch with how pilots and sailors navigated across oceans back in 1931.[7]

But not everybody enjoys the same things. If you would like to do your navigation calculations with an fx260, and use the GPS on your iPhone, you go right ahead. It makes perfect sense for you to do so.

Introduction

Part I of this manual outlines some of the formulas used in navigational mathematics that are of particular interest to small boat sailors. It is divided into two groupings: formulas of greatest use in coastal navigation, and formulas used in offshore navigation.

Part II details how these formulas may be solved using the Mark 1 Navigator's Slide Rule. This part is divided by slide rule functions (multiply, divide, squares, trig) so that you can gain facility in using the rule in a step-by-step fashion.

Bowditch (see Bibliography at back) is the gold standard for the navigational mathematics adopted for this manual. Since Bowditch uses American-customary units (feet instead of meters; inches instead of cm), this user manual also uses American-customary units.

I am deeply obliged to several members of NavList (www.fer3.com/arc) for suggestions both on the design of the rule itself, and for thoughtful editing of this user manual. In particular, Gary LaPook coached me regarding the Bygrave equations and rhumb line navigation.

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[6] Though sextant use is not quite as out-of-date as you might think. Here is a TV commercial that Chevron made in 2017 to reassure people that they didn't just depend on GPS and computers to navigate their super-tankers. They keep their sextant skills current, to provide an independent, redundant navigational method to ensure the safety of the vessel in the event that "bad things" happen: https://www.youtube.com/watch?v=5X9a-y8d0vQ

[7] https://en.wikipedia.org/wiki/Francis_Chichester#Air_pilot The mathematics for sight reduction/great circle calculation presented in this manual are fundamentally the same as what Chichester used. He, however, used a cylindrical slide rule, which has an even more arcane role in navigational history than the straight slide rule.

Any errors that remain are purely my own.

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§1. Abbreviations

Abbreviations used in various parts of this manual include:

AP	assumed position[8]
d	declination of geographic position of celestial object; also used of 'latitude of destination' in great circle calculations
D	distance in nautical miles
Dip	Dip is not an abbreviation, but a standard expression in celestial navigation. Expressed in minutes of arc.
DR	Dead reckoning position
GP	geographic position
h	height of eye above water level, in feet
H	height of object (e.g. light of a lighthouse, mountain peak) above water level, in feet; also used of 'meridian angle' in great circle calculations and sight reduction
Ho	height observed: height measured by sextant, after corrections for dip and refraction have been applied.
Hr	time in hours
Hs	height measured by sextant, prior to corrections for dip and refraction
S	speed in knots
SOG	speed over the ground, in knots
LHA	local hour angle
LOA	length overall
LWL	length of vessel at the waterline, in feet
T	time in minutes
VMG	velocity made good – the velocity in the direction of your destination. If you are reaching or running, your SOG can be the same as your VMG. If you are tacking, VMG will be less than SOG.
Z	azimuth angle
Zn	azimuth
γ	some angle (pronounced gamma)
Δ	a difference in angle or distance (pronounced delta)
θ	some angle (pronounced theta)

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[8] If you are not quite sure what this means, see §25.

Part I: Navigational Mathematics

Once you become comfortable with using your slide rule, you may choose to print out the sheets of paper that comprise Part I, fold them up, and stick them inside your copy of the Nautical Almanac, to give you a concise reminder of things you know how to do.

§2. Conversions

1 knot = 1.852 km/hour

1 foot = 0.305 meters

1 nautical mile = 1.852 km

1 US gallon = 3.785 liters

1 fathom = 6 feet = 1.829 meters

The Mark 1 Navigator's Slide Rule has "gauge points" (see §31 and §32) to facilitate the conversion of liters to US gallons, and meters to feet.

Coastal Navigation

§3. Chart Scales & Distance

Nautical Miles Per Inch = Reciprocal of Chart Scale ÷ 72,900

§4. Speed, Time and Distance

Work out problems in speed, time, and distance by using the D-scale to read distance on, and the C-scale to read time on.

§5. Calculate Boatspeed with a Dutch Log

In the event your knotmeter fails, you can determine your boatspeed by putting the right index over the DL gauge point at .5925. Move the cursor over the C-scale value of the LOA of your boat in feet. Without moving the cursor further, adjust the slide until the number of seconds that passed for a chip of wood to move from the bow to the stern is under the cursor. Read the boatspeed under the index.

§6. Solving the Current Triangle

Position the angle of the current off the rhumb line course, measuring 0° to 90° off the bow, or 0° to 90° off the stern, on the S-scale over the top of your boatspeed on the D-scale. Slide the cursor on the D-scale over the value of the drift of the current on the D-scale. Read the compass correction from the S-scale.

Without moving your slide, shift the cursor over to the angle-of-current-off rhumb line minus the correction on the S-scale, if the current is on the bow, or plus the correction if the current is on the quarter. Read the SOG from the D-scale.

§7. Fuel Consumption

At 75% of maximum revolutions on your diesel engine:

Gallons per Hour = Horsepower x 0.055

§8. Maximum Hull Speed

For a displacement, non-planing, hull:

$S = 1.34 \times \sqrt{LWL}$

§9. Distance to Visible Horizon

$D = 1.17 \times \sqrt{h}$

$D = 2.07 \times \sqrt{\text{height of eye in meters}}$

§10. Calculating Intermediate Tide Heights

Use the quarter-tenth rule to construct a sine curve of the tide heights over time.

§11. Tacking Downwind

For given wind/sea conditions, plus sail configuration with your specific hull shape, calculate "distance factor" (DF), "speed factor" (SF) and "time factor" (TF) for each possible course you are considering.

$DF = \frac{\cos(\gamma)}{\cos(\theta)}$

$SF = \frac{S\theta}{S\gamma}$

$TF = \frac{DF}{SF}$

Fill in this table, and select the smallest TF value to determine your ideal course.

Course to Dest. relative to dead downwind γ	Course sailed relative to dead downwind θ	Distance Factor DF $\cos(\gamma)/\cos(\theta)$	Speed on course θ $S\theta$	Speed Factor SF $S\theta/S\gamma$	Time Factor TF DF/SF	Ideal Course
		1.000		1.000	1.000	

§12. *Tacking Upwind*

Use the same technique as with sailing downwind...except that you accelerate more as you point lower, rather than accelerating as you point higher.

§13. *Distance Off by Vertical Angle Measured Between Sea Horizon and Top of Object Beyond Sea Horizon*

Constants for refraction and curvature of earth = 0.0002419 and 0.7349
Height = difference in feet between height of object and eye of observer
Distance is in nautical miles

$$\text{Distance} = \sqrt{[\tan(\text{angle}) \div 0.0002419]^2 + [\text{height} \div 0.7349]} - [\tan(\text{angle}) \div 0.0002419]$$

As long as the angle is greater than 0° 34.4’, you can calculate this on your slide rule. If the angle is less than that, you will need to use a trig-enabled calculator.

§14. *Distance Off by Two Bearings and the Run Between*

$$\frac{\sin(\text{difference between bearing 1 and bearing 2})}{\text{distance run}} = \frac{\sin(\text{bearing 1})}{\text{distance from object when bearing 1 was taken}} = \frac{\sin(\text{bearing 2})}{\text{distance from object when bearing 2 was taken}}$$

§15. *Distance to Radar Horizon*

$$D = 1.22 \times \sqrt{h}$$

…where h is the height of the antenna in feet.

Offshore Navigation

§16. Dip

The proper, accepted equation for calculating dip is:

$\text{Dip} = 0.97 \times \sqrt{h}$

However, many navigators simply use:

$\text{Dip} = \sqrt{h}$

§17. Dip for Short Horizon

When taking a sextant sight and finding an island or another vessel lies directly "underneath" the celestial object you are sighting, you must use a "short horizon" calculation for dip if that island/vessel is closer than the natural horizon (see §9). You must know (or estimate) the distance D to the waterline of that island/vessel.

$\text{Dip}_{\text{Short Horizon}} = (0.416 \times D) + (0.566 \times (\frac{h}{D}))$

This short-horizon equation may also be used if you are practicing sextant sights on the shore of a lake, as long as you know the distance to the opposing shore.

§18. Scope of a Given Universal Plotting Sheet

Usable latitude range in minutes = 34 ÷ sin(Latitude)

§19. Sight Reduction/Great Circle Route Calculations

Calculating a great circle route and performing sight reduction on a sextant sight are fundamentally both solutions to the navigational triangle.

When doing sight reduction rather than route planning:

- Substitute the coordinates of the GP of your celestial object for the destination, and…
- Omit Step 6 from the following worksheet.

Great Circle Calculations/ Sight Reduction

If using for celestial SR, "Destination" = the GP of the celestial object.
Latitude Destination = Declination of object.
Longitude Destination = GHA.

My Longitude = ____ (E/W) ________°

Longitude Destination = ____ (E/W) ________°

Meridian Angle = ____ (E/W) ________°

DATA INPUTS: Meridian Angle < 90°	Differences Where Meridian Angle > 90°
Meridian Angle (t) = ____ (E/W) ________°	180°- Meridian Angle = (t) = ____ (E/W) ________°
My Latitude (L) = ____ (N/S) ________°	
Latitude Destination (d) = ____ (N/S) ________°	

1. tan(d) ÷ cos(t) = tan(W)

   W = ________°

2. Use [+W] if d has same sign as L.
   Use [-W] if d has opposite sign as L.

   (90° - L) ± W = X

   X = ________°

   Where Meridian Angle > 90°: 2. (90° - L) - W = X

   X = ________°

3. Ignore the sign of X (i.e. -60 = 60).
   If X < 90°, then X = Y
   If X > 90°, then 180 - X = Y

   Y = ________°

4. [cos(W) ÷ cos(Y)] * tan(t) = tan(Z)

   Z = ________°

5. cos(Z) * tan(Y) = tan(Hc)

   Hc = ________°

   Zn = ________°

   Convert Hc from decimal deg to deg/min if plotting a celestial position.

   Hc = ____° ____'

6. (90° - Hc) * 60 = Distance in nm

   Distance = ________ nm

Azimuth Rules for Step 5

Meridian angle (t)	1° to 179° W	1° to 179° E
L is in North Latitude		
If d or W > L	Zn = 360 - Z	Zn = Z
if d contrary or W < L	Zn = 180 + Z	Zn = 180 - Z
L is in South Latitude		
If d or W > L	Zn = 180 + Z	Zn = 180 - Z
if d contrary or W < L	Zn = 180 + Z	Zn = 180 - Z

Equation 5 works for sight reduction of all sextant sights, as well as for great circle route calculations. Equation 6 works where the distance to be travelled is no more than 1/4 the circumference of the earth away from starting point.

§20. Error Ellipse in Celestial Fix

Long Axis of Ellipse = Short Axis of Ellipse ÷ $\tan(\theta/2)$

θ should always be less than 90°. Treat a θ of 160° as $\theta = (180° - \theta)$, or $\theta = 20°$.

Rhumb Line Route Calculation

1	Destination latitude
2	Destination latitude in decimal degrees
3	My latitude
4	My latitude in decimal degrees
5	Difference between destination latitude and my latitude in decimal degrees (i.e. ΔL)
6	Meridian angle between my longitude and destination longitude (i.e. t)
7	Convert t into decimal degrees.
8	π * t in decimal degrees
9	(destination latitude in decimal degrees ÷ 2) + 45
10	Convert value from line 9 into degrees and minutes
11	Ln of tan(result of line 10)
12	(my latitude in decimal degrees ÷ 2) + 45
13	Ln of tan(result of line 12)
14	Subtract line 13 from line 11
15	(result of line 14) * 180
16	Divide value from line 8 by value in line 15
17	atan(value of line 16) = Z of course as __ ___° ___' __ (e.g. N 10° 22' W)
18	Rhumb line course Zn
19	60 * (ΔL from line 5) ÷ cos(Z from line 17) = rhumb line distance in nm

§22. Plotting Longitude

Using a standard VP-OS plotting sheet,[9] you may use dividers with the scale printed in the lower right corner. However, you might find it faster to use the scale of 60' to 0' to 60' engraved on the reverse of the Mark 1.

Position the rule along the central latitude line, then rotate the rule from horizontal by an amount equivalent to the latitude of your assumed position.

§23. Compass Delta

Sin(Azimuth) = sin(declination) ÷ cos(latitude)

If you use the:

- Sun, observe when the lower limb is 2/3 of the sun's diameter above the visible horizon.
- Moon, observe when the upper limb is right on the visible horizon.
- Star/planet, observe when the object is a little more than a sun diameter above the visible horizon.

§24. Trigonometry and Mental Math

Whenever in doubt about which "T" scale to use in reading your answer, convert all your trig functions into numeric values, do your multiplication or division, and get a range for your answer. Then look at the right edge of the various T scales for the short notes that document the value range of each scale.

[9] i.e. the VP-OS sheet published by Weems and Plath, or one that uses precisely the same scale.

Part II: Using the Mark 1 Navigator's Slide Rule

First Things

§25. Can You Learn to Navigate From This Manual?

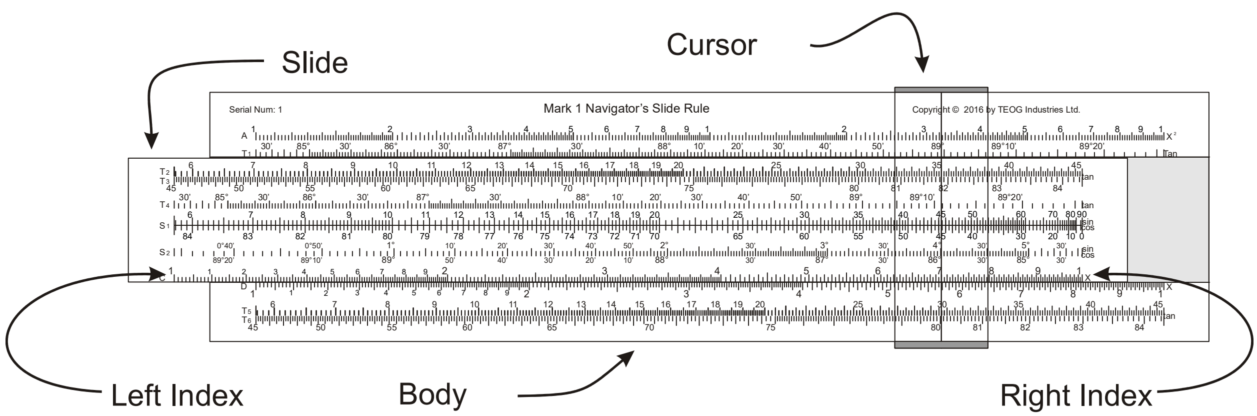
No.

This user manual is intended to orient you to using the Mark 1 slide rule to solve common navigational problems. It assumes that you *already* understand how to navigate, including how to use a sextant to determine your location.

§26. Parts of the Slide Rule

The slide rule is made up of three parts:

- Body
- Slide
- Cursor



There are scales of various sorts, labelled with alphabet letters along the left, together with a short description of what the scale does on the right.

The hash mark under the 1 on the left side of the slide is known as the left index. The hash mark under the 1 on the right side is known as the right index.

There are three gauge points marked immediately beneath the D scale. Two of these are to simplify doing common conversions: liters to gallons and meters to feet. See §31 and §32. The third is for use with a Dutch log (§33).

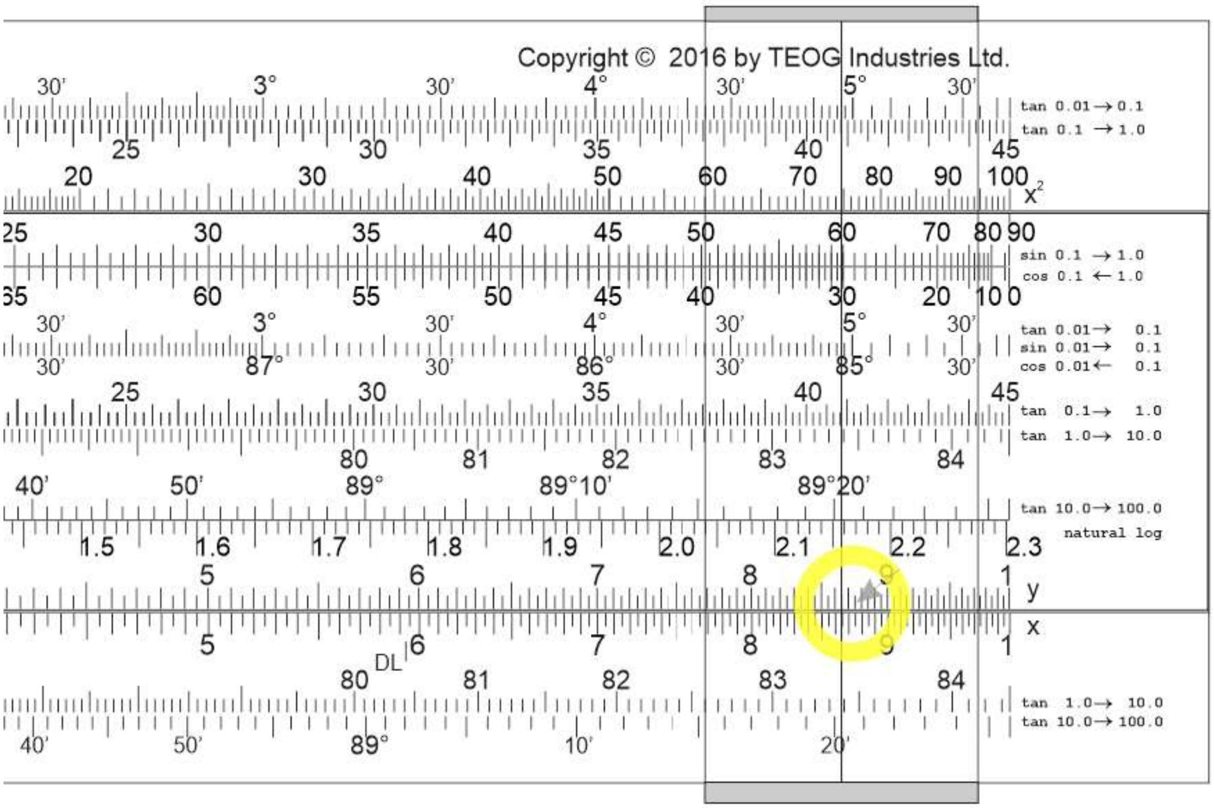
§27. Reading the C & D scales

The numbers on the C and D scales range from 1, to 2, to 3, etc…on up to another 1.

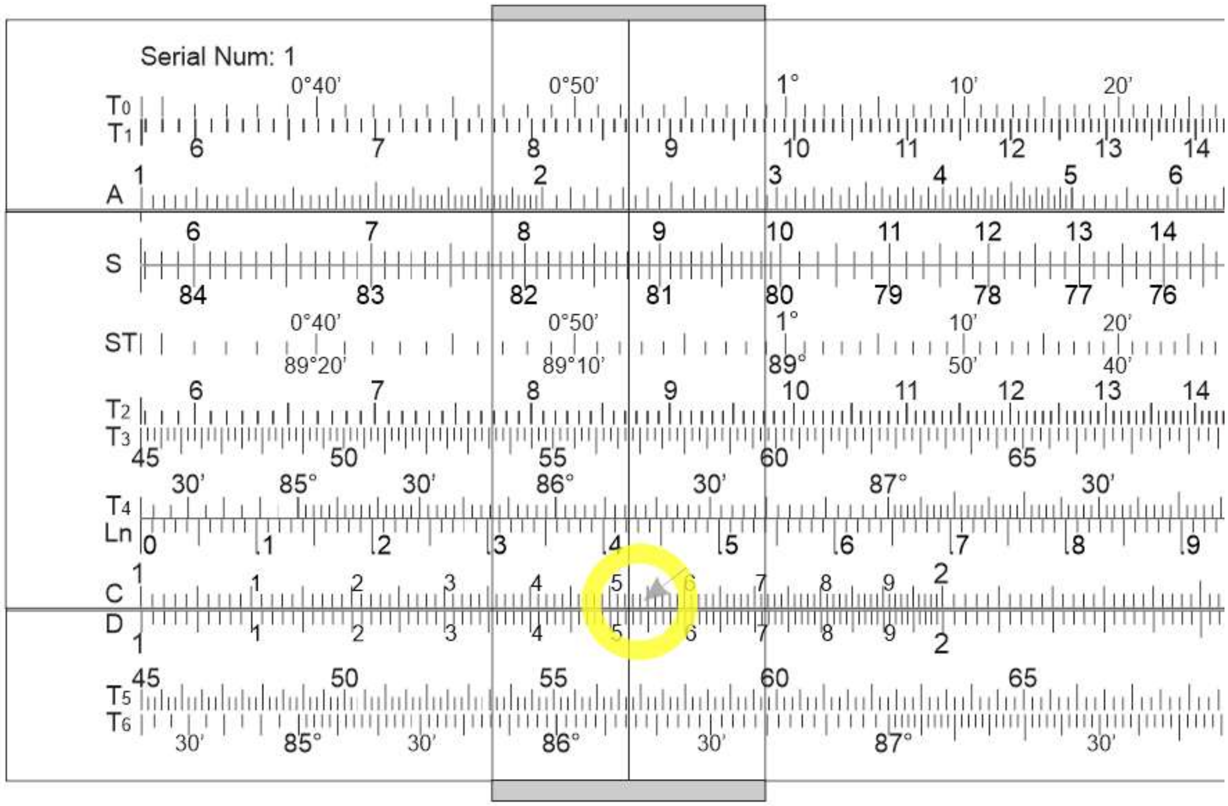
Depending on which part of C or D you are looking at, you can read numbers 3 or 4 digits long.

Another way of saying this is that "You can read answers on a slide rule with up to 3 or 4 significant digits."

See below for a result on the right hand end of the scale, where numbers are closer together. Here, we can read an answer to three significant digits. The cursor is positioned over 865.



The numbers are further apart on the left hand side of the scale, and we can read results up to 4 significant digits. In this next illustration, the cursor is positioned over 1522.



It is up you as a user to determine whether a particular sequence means 1.522, 15.22, 152.2, or 1,522. That is, you must work out the position of the decimal place on your own.

If you have never used a slide rule before, working out the decimal places can seem daunting. You must do a certain level of basic calculation in your head so you know roughly how big the number is you at which you are going to arrive.

For instance, if you want to calculate:

13.82 × 11.02

…then you know the result is going to be rather more than 10 × 10 and rather less than 20 × 20. That is to say, the answer will be more than 100 and less than 400.

So when you work it out on your slide rule and see that the digits of the answer are 1522, you know you can position the decimal point properly, and that the answer is 152.2.

§28. Speed, Time and Distance

If you have done speed, time and distance calculations with an electronic calculator, you have undoubtedly used these three equations, where D = distance in nautical miles, S = speed in knots, and T = time in hours.

D = S x T T = D ÷ S S = D ÷ T

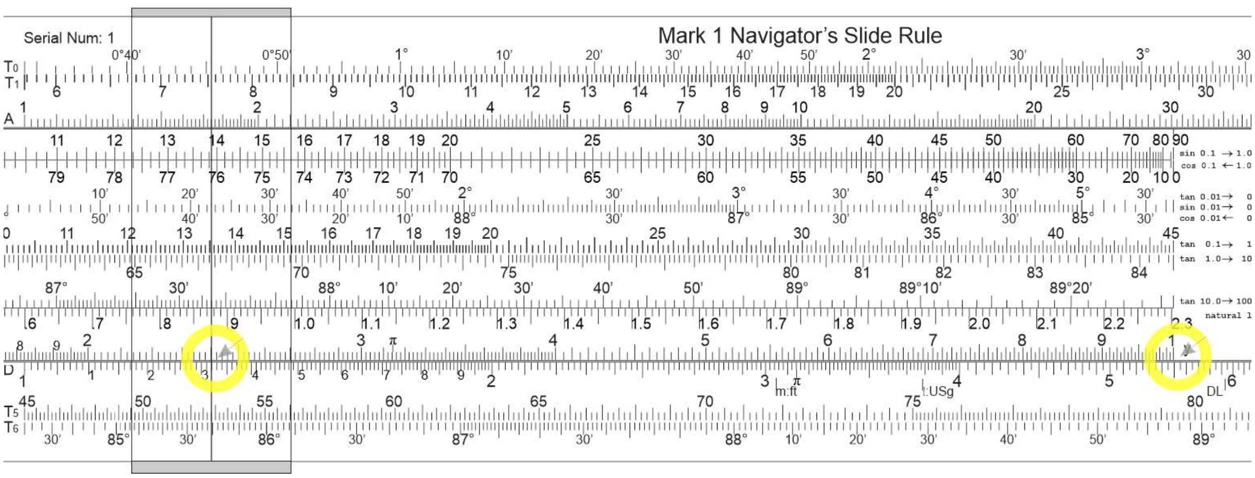
But as easy as these are to work, you can solve these sorts of problems even easier with a slide rule by using the C-scale to represent time and the D-scale to represent distance.

§28.1 Knowing Time and Distance, Solve for Speed

You are sailing from Alexandria, Egypt to Malta, and your boatspeed has been consistently around 6.3 kn. You know there is a counter-clockwise current in the eastern Mediterranean, up the coast of Palestine, around the south coast of Turkey, over to Greece and down to Libya, and back along to Egypt. [10]

You have a steady wind out of the NW, so you can make good your course for Malta…but you expect that you are taking this current right on the nose. What you don't know is just how strong that current is at this time of year. What is your SOG?

You determine that over the first 24 hours of sailing, you have travelled 132 nm. Position 24 on the C scale over the top of 132 (nautical miles per 24 hours) on the D scale and look at the right index to read your speed per 1 hour: 5.5 nm. And 5.5 nm in 1 hour = a SOG of 5.5 kn.



You can now infer that the current is setting against you at the rate of 0.8 kn.

§288.2 Knowing Speed and Distance, Solve for Time

You know that Malta is 819 nm from Alexandria. Assuming you maintain your current speed, how long will it take you to arrive, from beginning to end?

You know you have travelled 132 nm in 1 day. To know how many days it will be before you arrive, set the left index over 132 on the D scale. The index value of 1 represents the time value of 1 day.

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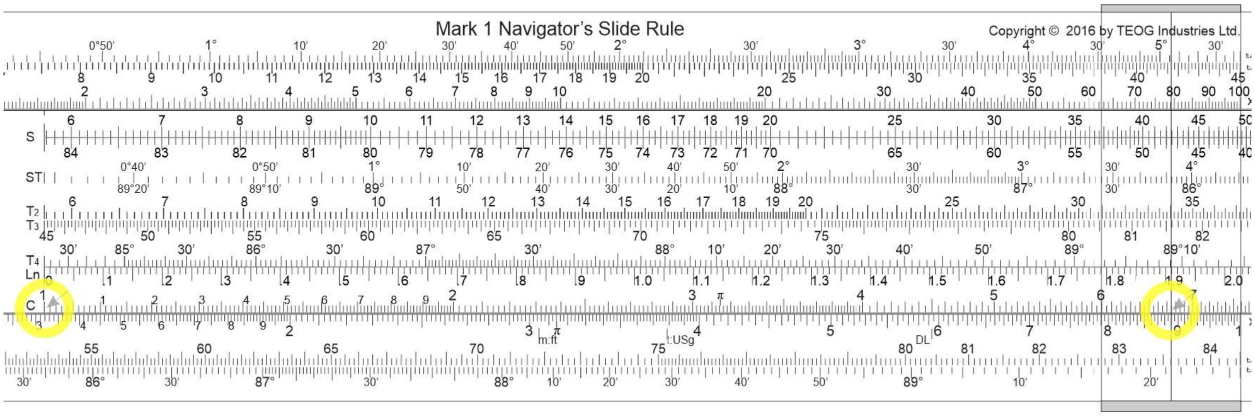
[10] This current is driven by the output from the Nile, and further influenced by surface-level winds.

You can treat time on the C-scale in terms of hours, days, or minutes. It doesn't matter what unit of time you use…as long as you carry through with those units for your entire calculation. If your time input is in minutes, then your answer needs to also be related to minutes.

Move the cursor over until it is over the top of 819 on the D-scale (representing distance). You can read off the C-scale that if you maintain your current course and speed, you will arrive in 6.2 days, or roughly 6 days and 5 hours.



While you have your slide rule positioned like this, you can project your DR distances from Alexandria on a day by day basis. After 2 days (read on the C-scale), you will have travelled around 264 nm (read on the D-scale). After 3 days, 396 nm. After 4 days, 528 nm. 5 days, 660 nm.

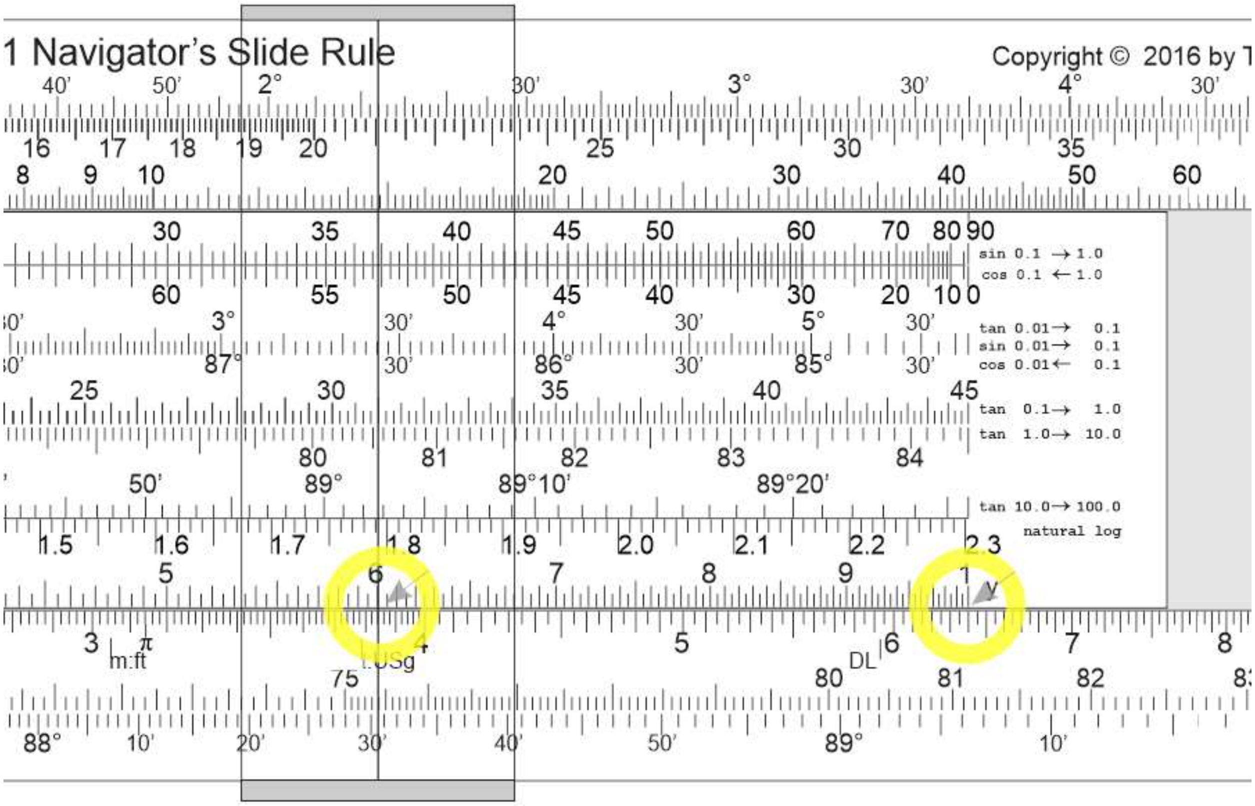
In this particular case, you can work out DR positions with your slide rule much faster than you can with an electronic calculator.

§288.3 Knowing Speed and Time, Solve for Distance

You have gotten out east of the Greek peninsula, and you have left that head-current in your wake. However, the wind has backed through north, to northeast and you find that you have to tack back and forth to make your course to Malta.

You are on a starboard tack and making 6.4 kn SOG just as you go off-watch. You would like to hold this tack, but you want to stay clear of the Syrtis Shoals off the coast of North Africa. The current watch will be on deck for 6 hours. How far will your vessel travel in 6 hours?

Position the right index over 6.4 on the D-scale, to represent 6.4 knots (i.e. 6.4 nm in 1 hour). Move the cursor over the 6 on the C-scale to see how far you will go in 6 hours: 38.4 nm (read on the D scale).



This brings you within 5 nm of a danger line you have defined for yourself. You know that you will sleep better down below if the vessel never comes closer than 12 nm from your danger line. To determine how long to hold on the current tack to travel 31.4 nm, without moving the slide at all, adjust the cursor across to 31.4 on the D-scale. Read 4.9 hours off the C-scale.

Ask the watch captain to set an alarm for himself, assuming the boatspeed doesn't change too much one way or the other, to ensure that he comes about onto a port tack in 4 hours, 54 minutes.

§29. Reading Sines, Cosines and Tangents

This section is boring. I include it for completeness, but I recommend skipping to the next section to begin *using* your new slide rule. The best way to sort out scales is to learn by doing.

- Sine values from ~6° to 90° lie along the top of the S scale.
- Sine values less than ~6° lie along the top of the ST scale.
- Cosine values from 0° to ~84° lie along the bottom of the S scale.

- Cosine values greater than ~84° lie along the bottom of the ST scale.

- On the slide, tangent values from ~6° to ~89° are on the T_2, T_3 and T_4 scales. The equivalent of these scales on the body of the rule are found in T_1, T_5 and T_6.

- The one wild card relates to tangent values less than ~6°. On the body, these values are displayed on scale T0. But on the slide, to save space, these values lie along the top part of the ST scale. It is possible to combine sine and tangent values for small angles together because, within the limits of slide rule accuracy, the values are identical.

§30. Solving the Current Triangle

I have always enjoyed the story in *The American Practical Navigator* about the 31 year old Nathaniel Bowditch navigating his vessel into fog-bound Salem in 1803, while other masters were hove-to outside the harbor,[11] waiting on the fog to clear. While I would unquestionably use radar and GPS if I were attempting this today, I always aspire to be the *kind* of navigator who would be capable of this sort of feat with few or no electronic aids. And even *with* electronic aids, it is a fundamental skill to be able to solve the current triangle. That is, to determine what compass correction you need to make to cope with a given cross current when your vessel is travelling at a particular speed.

The classic approach to handle this problem is by drawing vector diagrams. There is a faster way, however. With a slide rule, one can solve a current-triangle problem a good bit more quickly than he can draw such a diagram. And if variable winds mean the boat speeds up or slows down (or if the set and drift of the current varies in a known fashion), a helmsman can generate a new heading in seconds without needing to go down to the chart table again to calculate his course.

In this approach, the angle of the current is always measured from the bow, from 0° to 90°, or from the stern, from 0° to 90°. And whatever course correction you make, whether the current is coming from ahead of the beam or abaft, will always be *into* the current.

Assuming your boatspeed is 6.5 kn, and your heading is 90° T, and the current is on your bow, ahead of the beam, coming in from 120° T at 2.3 kn, solve for the correction you need to cope with the current and make good your intended course…and then determine your SOG on that heading.

Set the angle of the current off the bow, 30°, on the S-scale opposite the speed of your vessel, 6.5 kn, on the D-scale.

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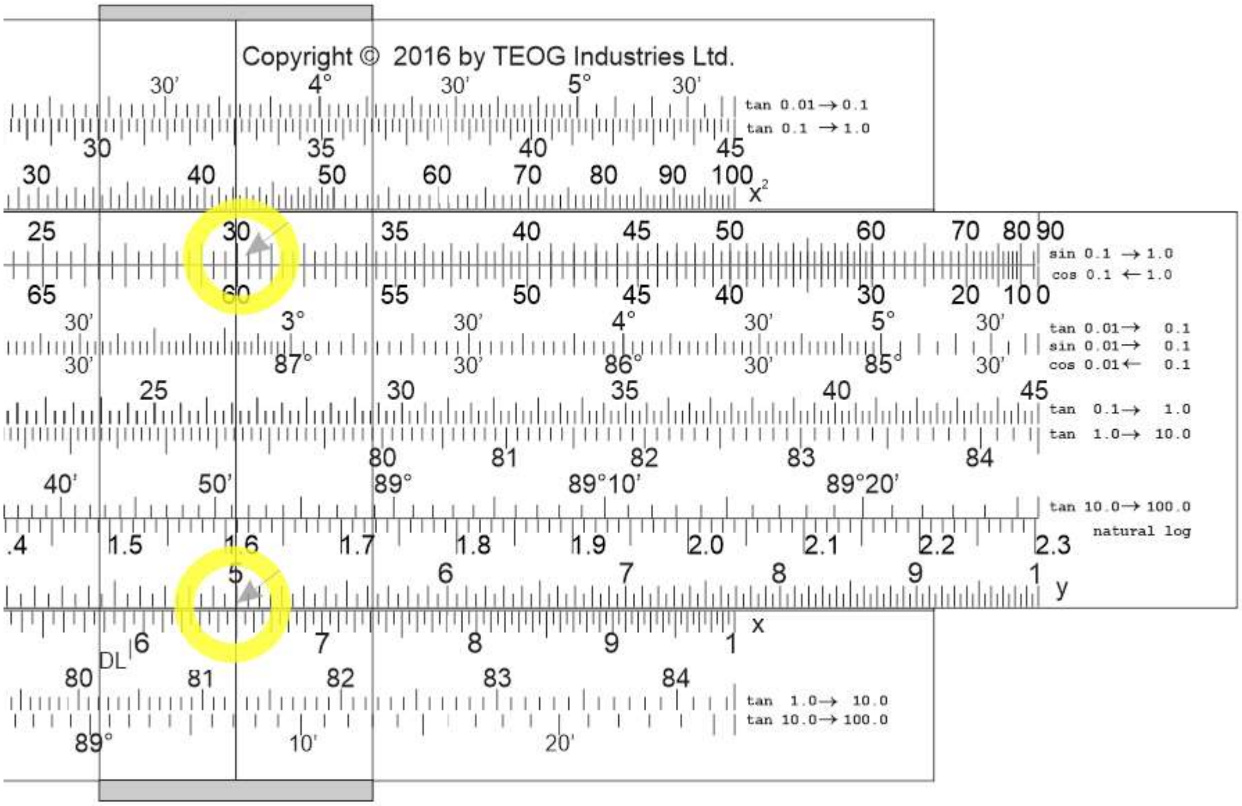
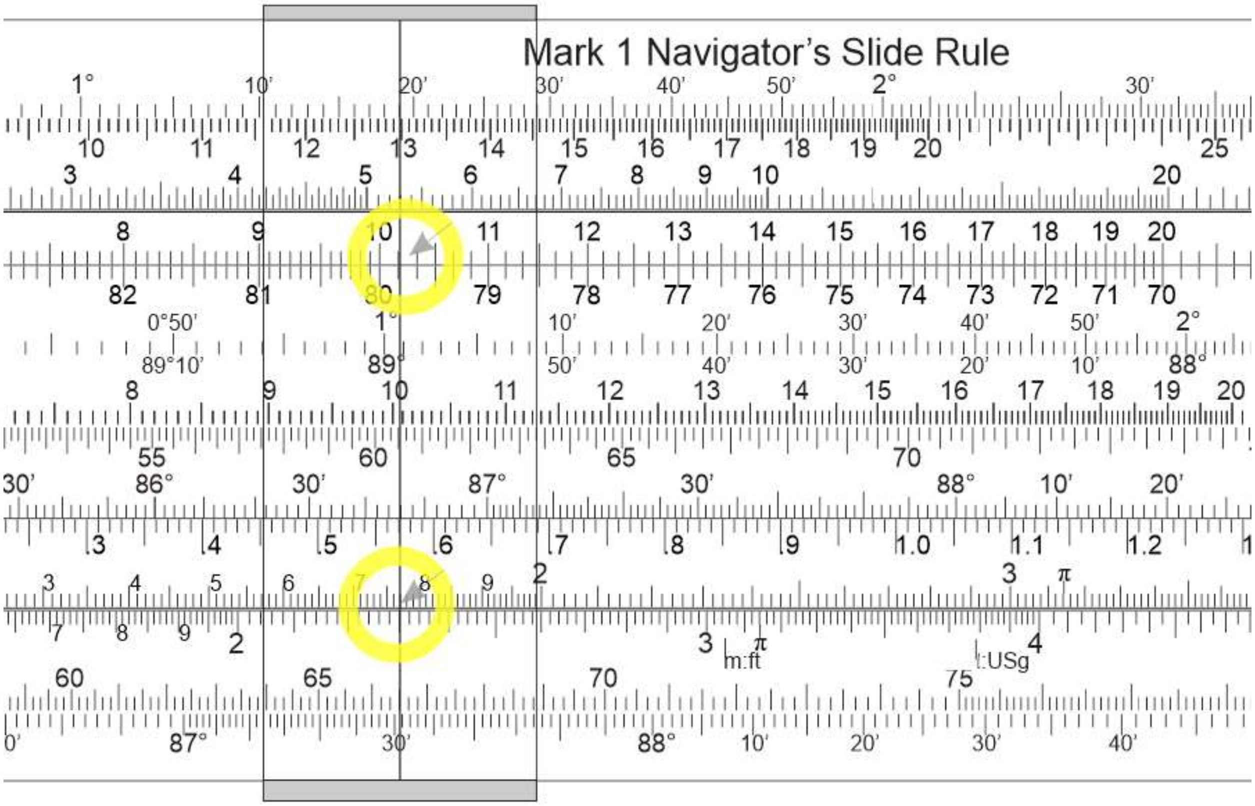
[11] Bowditch, p. iv.

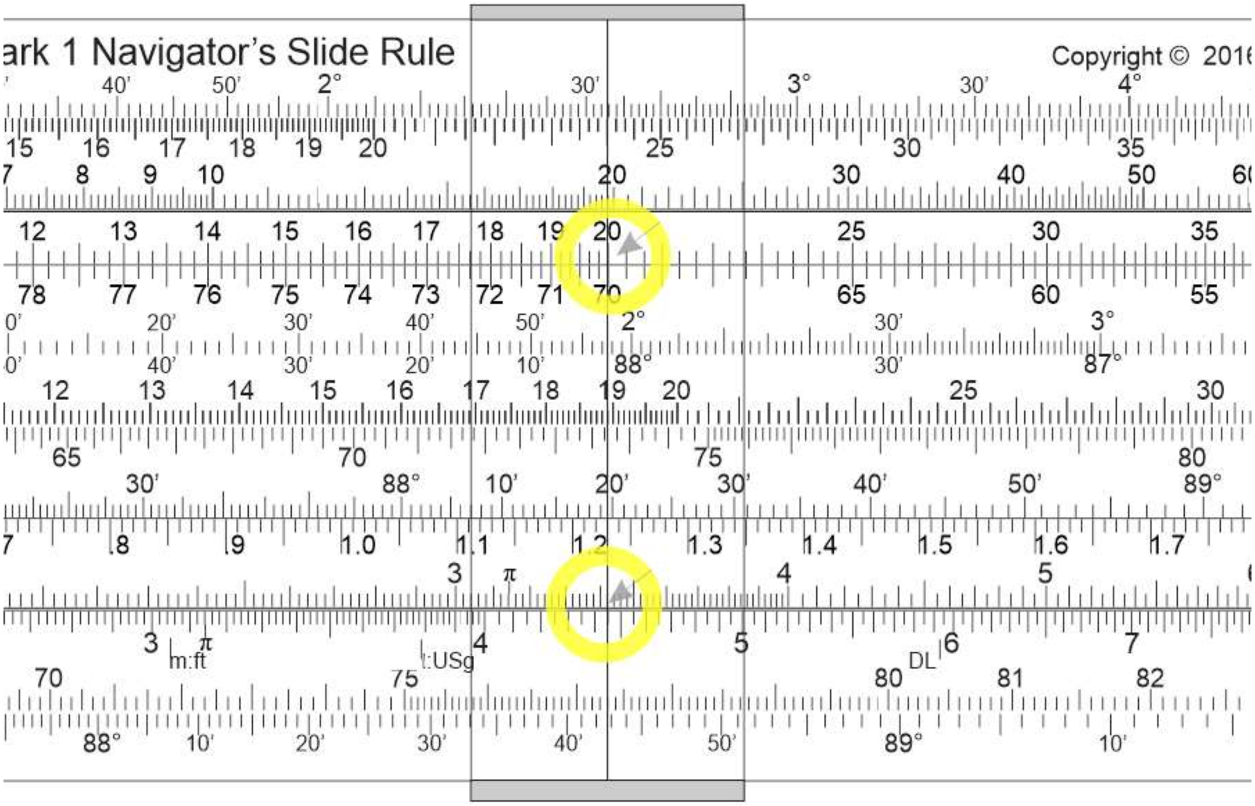


Move the cursor over the speed of the current, 2.3 kn, on the D-Scale. Read the course correction required off the S-scale: 10°.



Since you always make your correction into the current, you will need to steer a course of 100° to make good your intended course of 90° .

To determine your SOG, subtract the compass correction factor (i.e. 10°) from the angle of the current (i.e. 30°). In this case, without moving the slide of your rule, move the cursor over the top of 20° on the S-scale, and read your SOG on the D-scale: 4.5 kn.



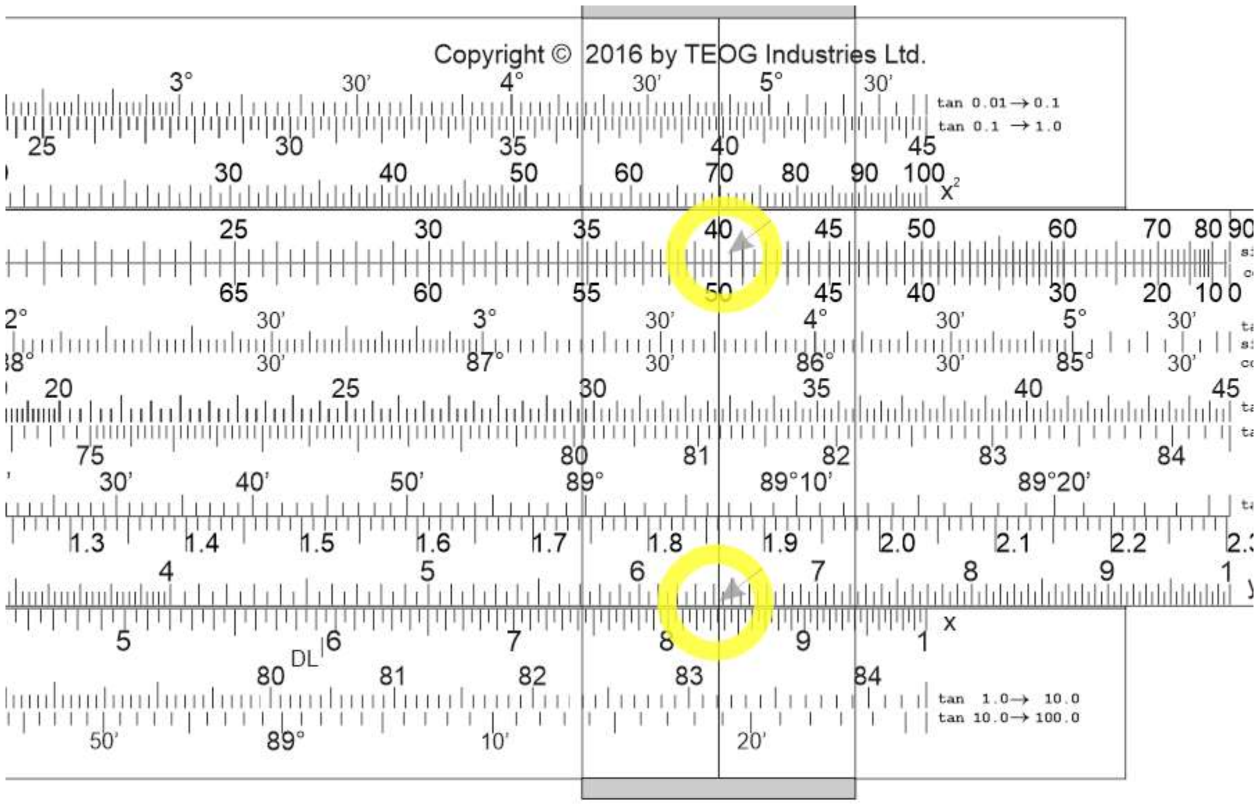
Work this problem again, but assume that the current is setting from 150° T…or 30° off your compass heading, but this time abaft the beam.

The current is still pushing you to port from your intended course. So your compass correction is still to steer 100° to make good your course of 90°. But because the current is abaft the beam instead of ahead of the beam, it will be increasing your SOG rather than decreasing it. So take the 10° compass correction factor and ADD it to the direction of the current off the rhumb line (i.e. current is 30° off the stern, so add 10° on the S-scale to 30°) and read the SOG off the D-scale. In this case, your SOG would now be 8.4 kn.[12]

[12] You can do these calculations on your hand calculator using the equations:

(sin(angle of current to rhumb line) * drift) ÷ boatspeed = sin(compass correction)
(Sin(angle of current to rhumb line ± compass correction) * boatspeed) ÷ sin(angle of current to rhumb line) = SOG

…but I find the slide rule method much faster.

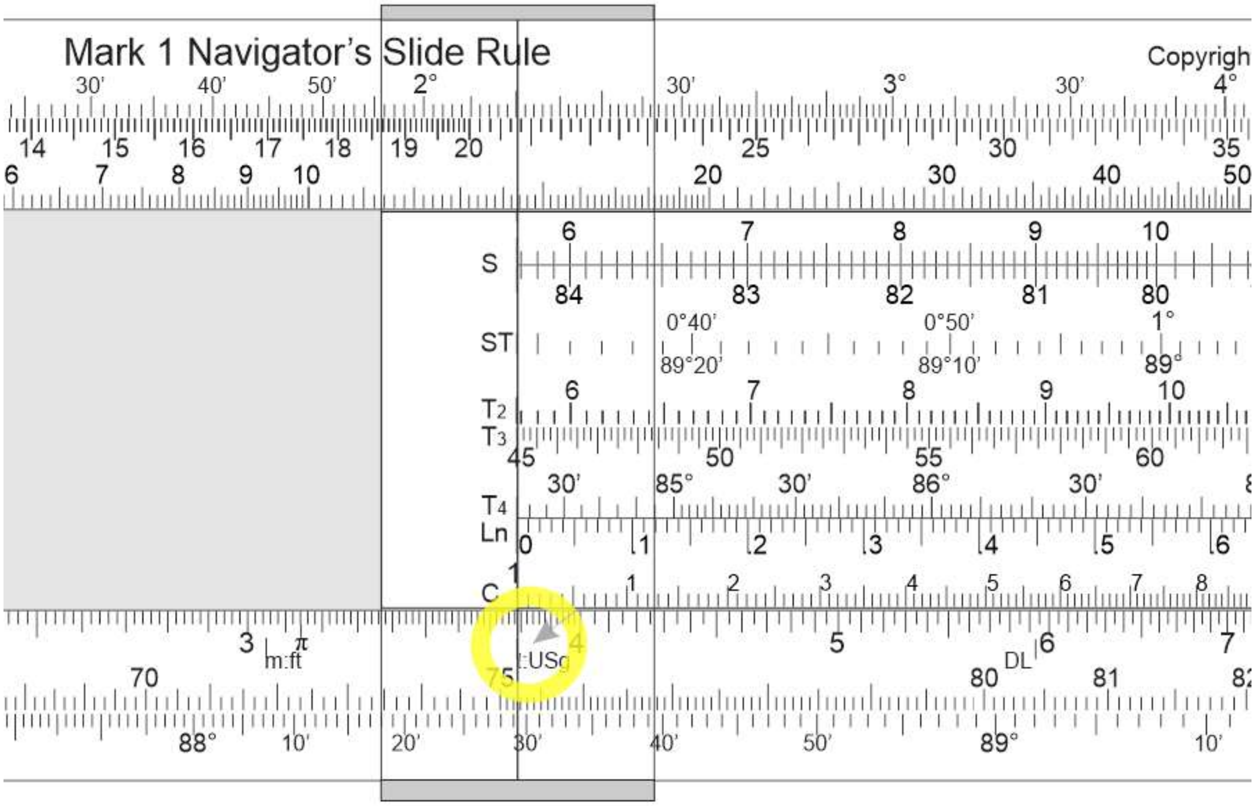


Using Gauge Points

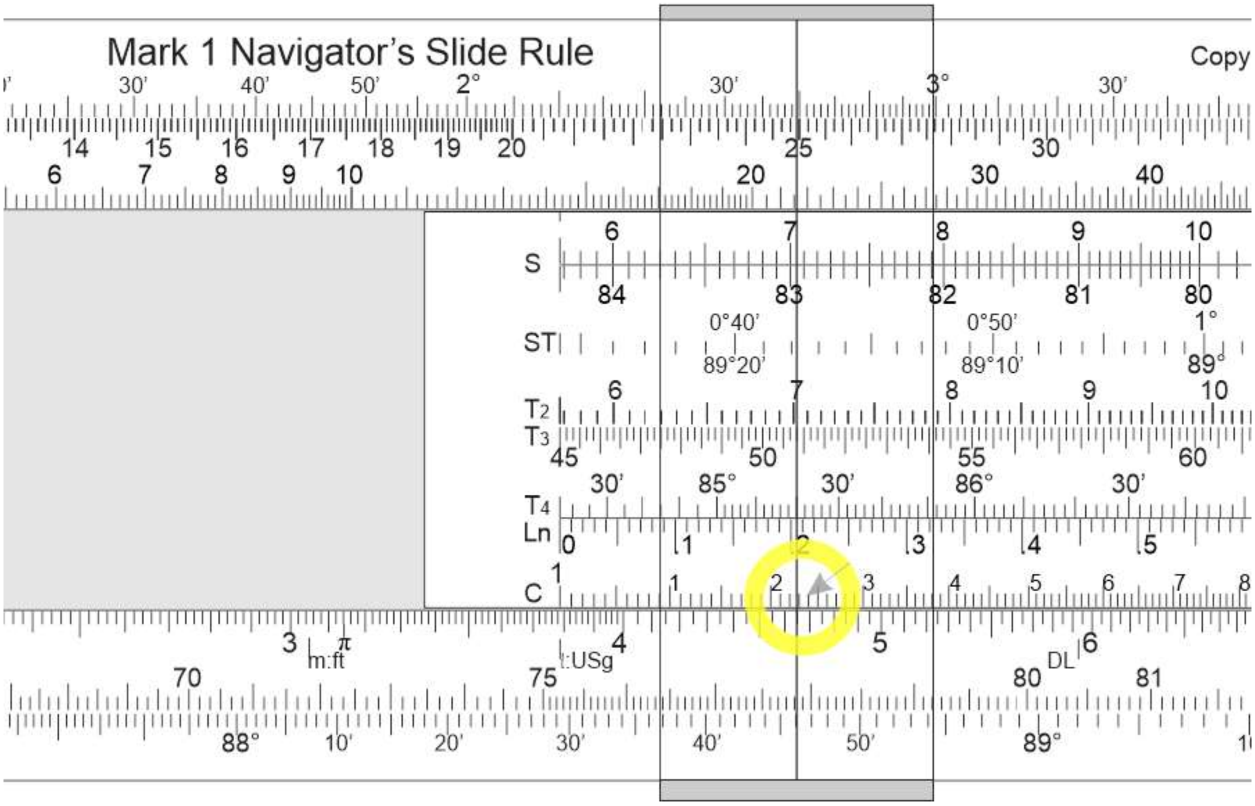
§31. Converting Liters to Gallons

While you normally cruise in United States waters, this summer you are sailing in Canada. You refill with diesel fuel at a marina, and you get 46.5 liters. How many gallons have you taken on?

Position the left index on the slide above the gauge mark labelled ℓ:USg.

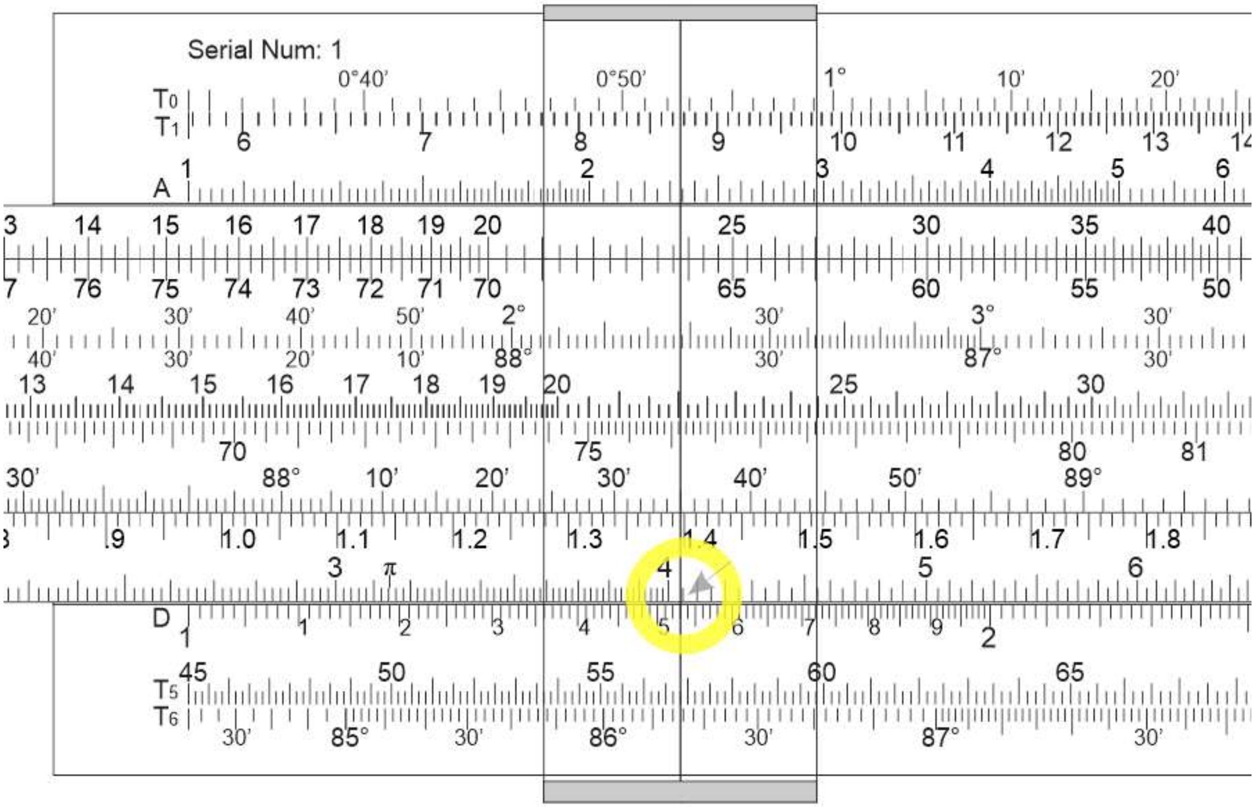


Then slide the cursor over until it is over 46.5 on the D scale. Read the gallons on the C scale: 12.28.



If the digits representing the liters you get are less than 385, such as 153 liters, then slide the right index over the gauge point. And again, slide your cursor over the number of liters on the D scale, and read the gallons on the C scale: 40.4.

You can slide your cursor along any value on the D scale, which now represents liters, and you will get the number of US gallons on the C scale.

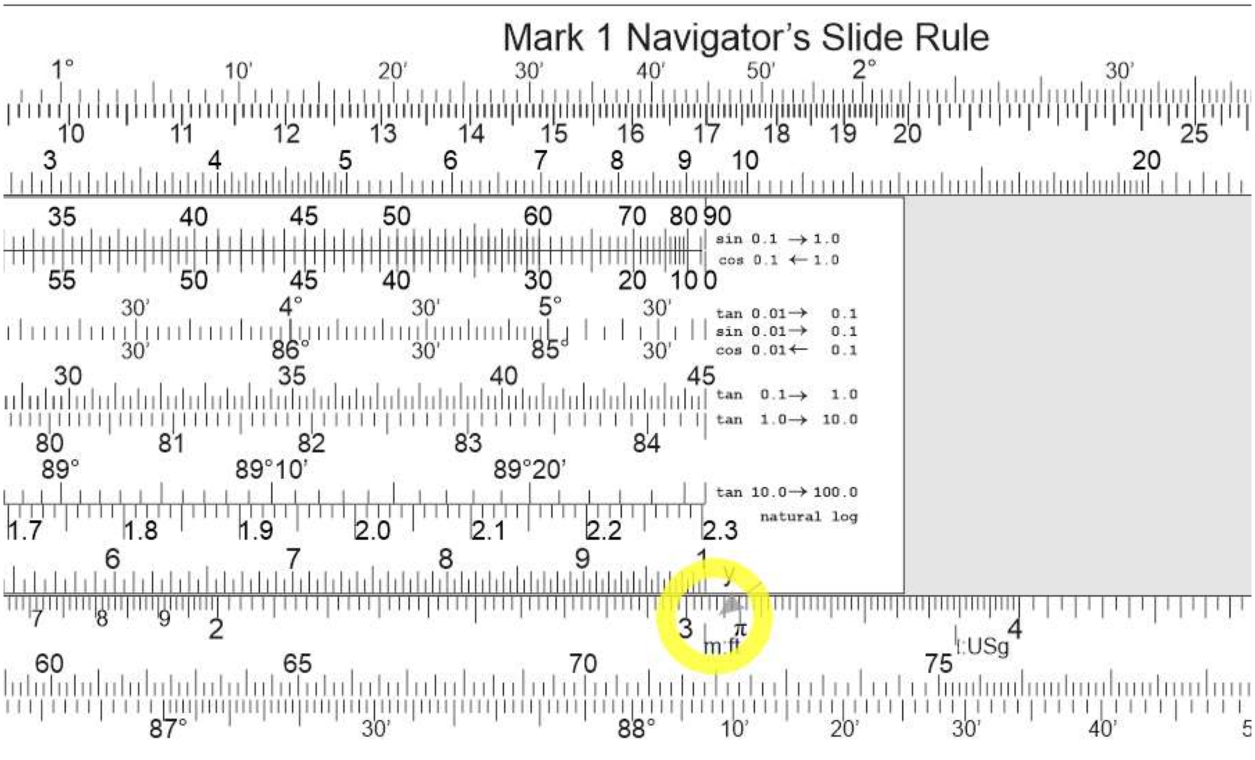


Getting the decimal point right. You know that a gallon amounts to approximately ¼th of a liter. You know that 1/4th of 100 liters would be 25 gallons. You know that 1/4th of 200 liters would be 50 gallons. So 1/4th of 153 liters will be between 25 and 50. So when you see 404 on the C scale, you know that it has got to be 40.4 (as opposed to 4.04 or 404).

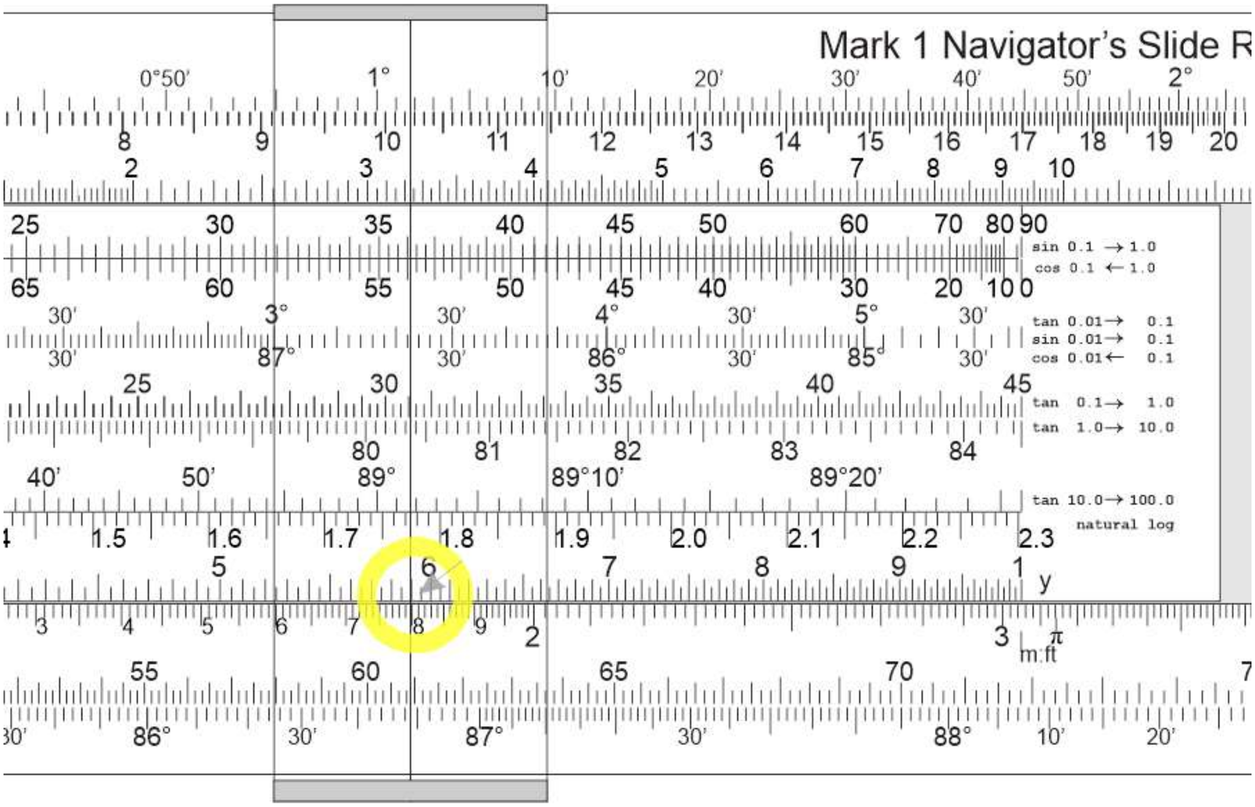
§32. Converting Meters to Feet

As you continue your cruise in Canada, you want to convert the depths you see on the chart – which are in meters – into feet. You see on the chart you are in 18 meters of water. What is the depth in feet?

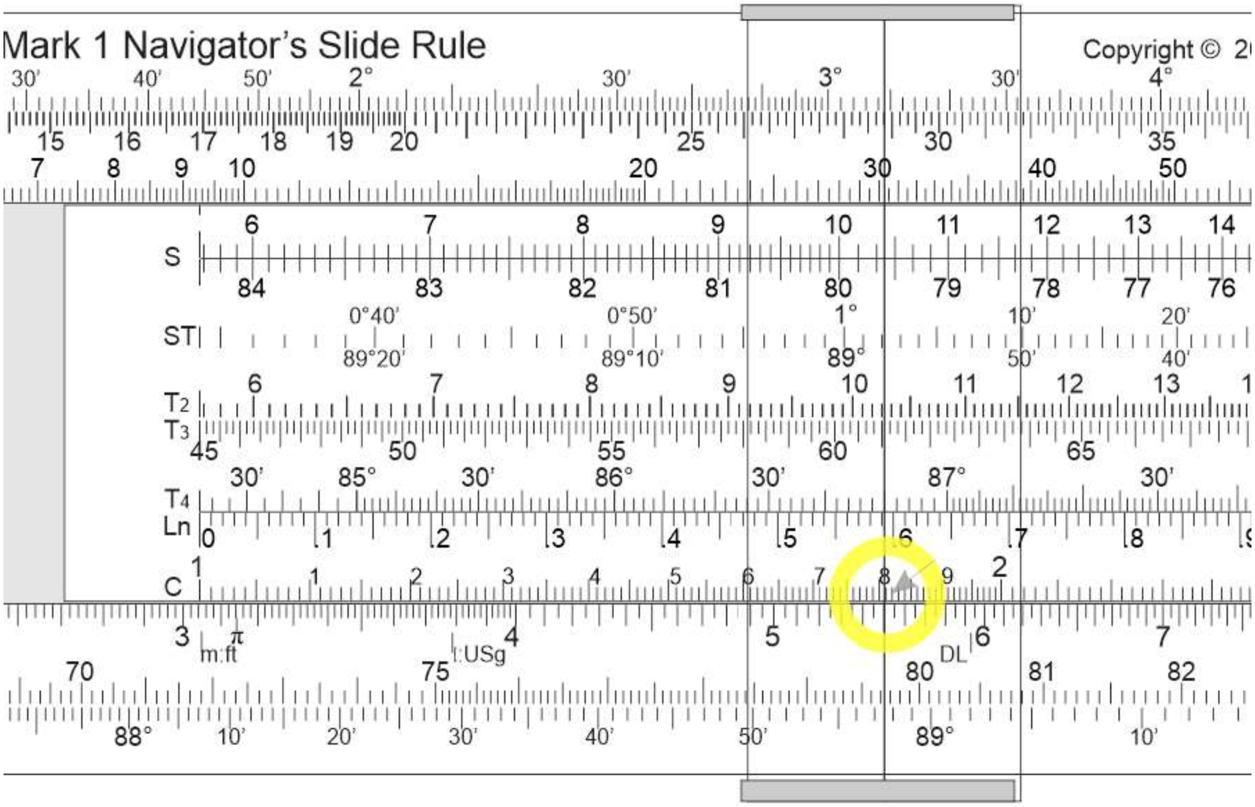
Move the right index over the gauge mark labelled "m:ft".



Move the cursor over the top of 18 on the D scale, and read the answer off the C scale:
59 feet.



If the initial numbers in the meters you want to convert are greater than 305, then slide the left index over the m:ft gauge point. So for instance, to convert 5.5 meters into feet, your slide rule will look like this:



5.5 meters on the D scale equate to 18 feet on the C scale.

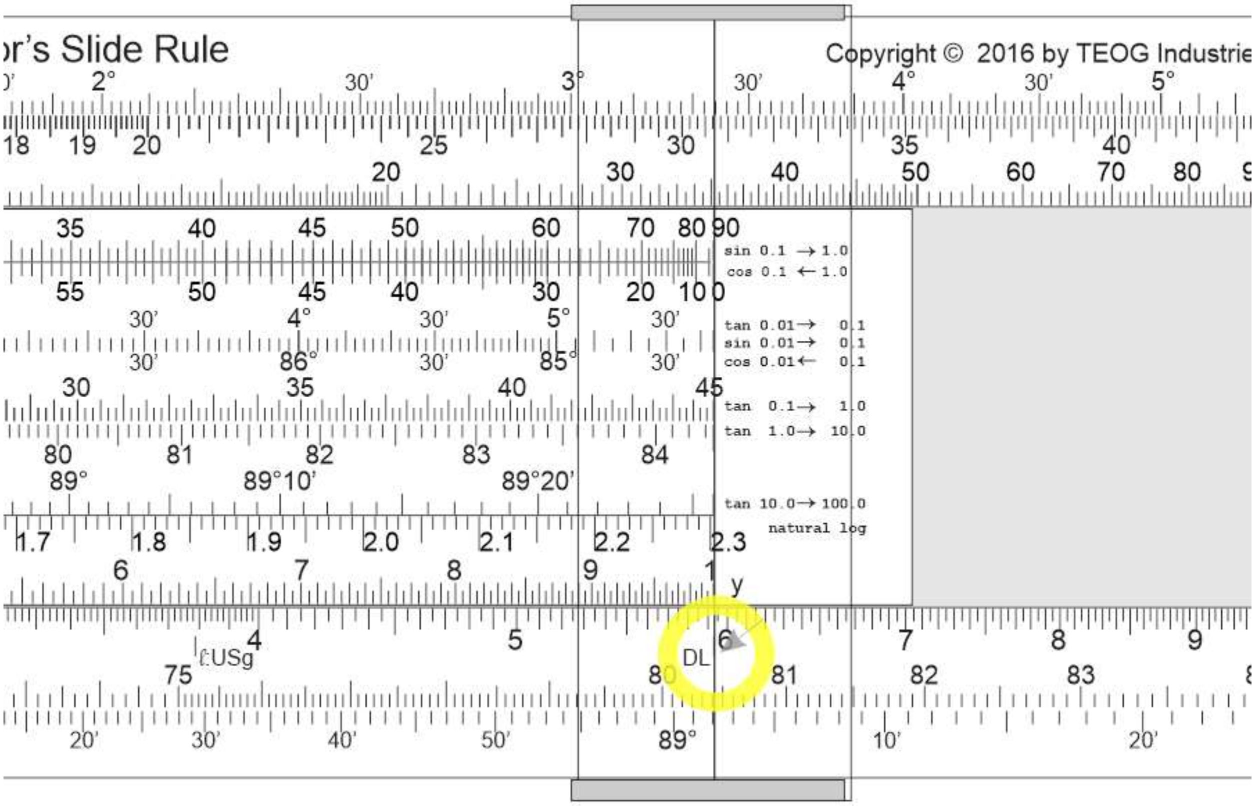
Getting the decimal point right. You know that there are approximately 3 feet in every meter. If you were interested in 5 meters rather than 5.5 meters, the answer would be in the neighborhood of 3 × 5 = 15. So when you see 18 as the first two significant digits in your answer you know that the answer is not 1.8 feet, nor is it 180 feet. The answer has got to be 18 feet.

§33. Calculate Boatspeed with Dutch Log

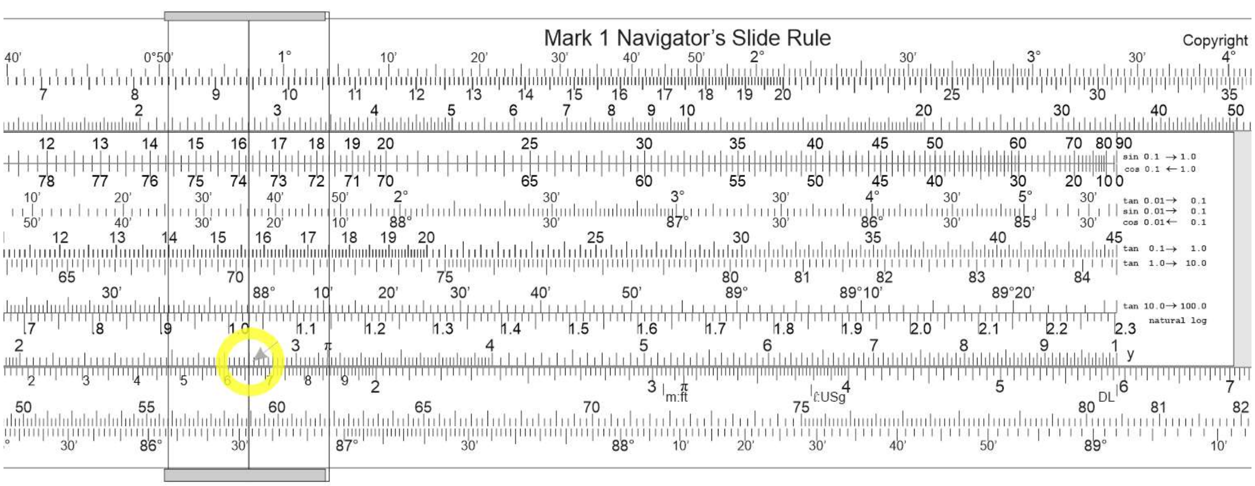
Probably once or twice out of every five times I charter a vessel, the knotmeter either stops working during the cruise, or never works at all. To do any sort of careful navigation, you need to know your boatspeed. One way of discovering this is with a Dutch (or Dutchman's) log.

Throw a piece of wood forward from the bow, and count the number of seconds that pass from the moment it comes even with the bow to the moment it comes even with the stern.

Position the right index over the gauge point at .5925 on the D-scale, marked by a "DL" (for "Dutch Log").

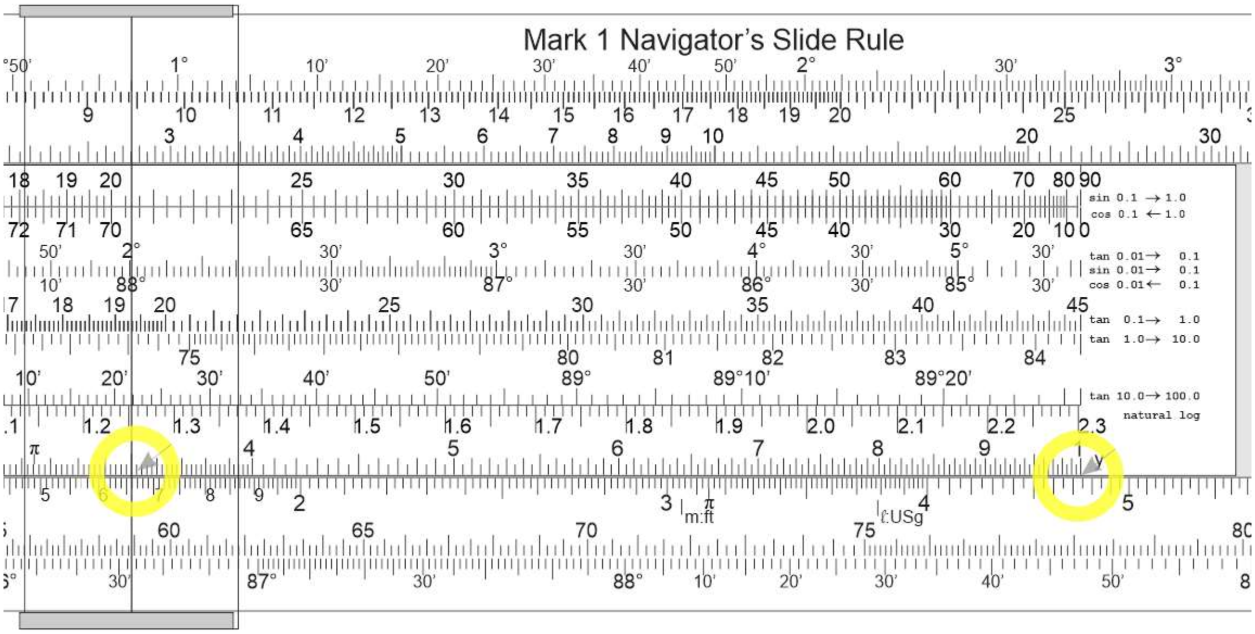


Slide the cursor out over the C-scale until it is over the top of a number that represents the LOA in feet. In this example, we will assume we are on a 28 foot boat.



Leave the cursor where it is, and slide the C-scale over until the number of seconds that passed for your wood (or other biodegradable object) to traverse the length of your boat is under the cursor. For this example, we will assume that it took 3.5 seconds for the chip of wood to travel the 28 feet length of your vessel.

Read the speed from the D-scale beneath the index.



In this example, the boatspeed would be ~4.7 knots.

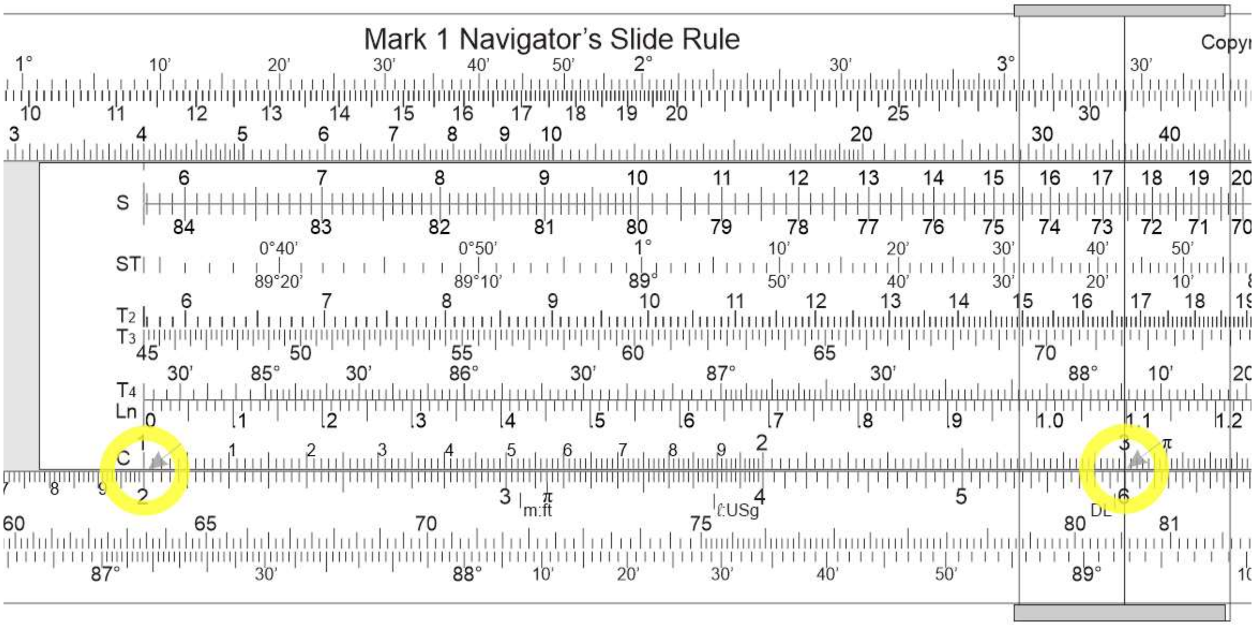
Of course, if you do not have a supply of wood chips to throw overboard, you can scan the surface for seaweed or debris and use that for timing rather than a piece of wood.

Multiply and Divide

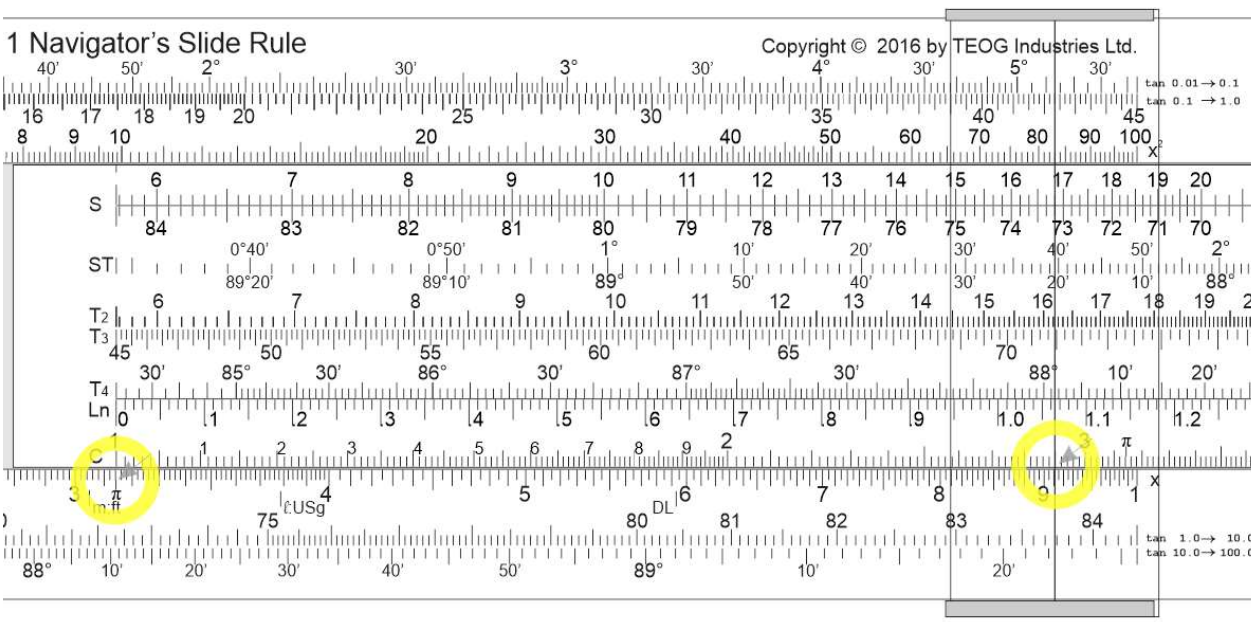
§34. Multiply

To multiply, position the left index over your first number on the D scale. Move the cursor along until it is over the second number on the C scale. The answer will be under the cursor on the D scale.

Multiply 2 × 3. Answer = 6



Multiply 3.141 (i.e. π) × 290 Answer = 911

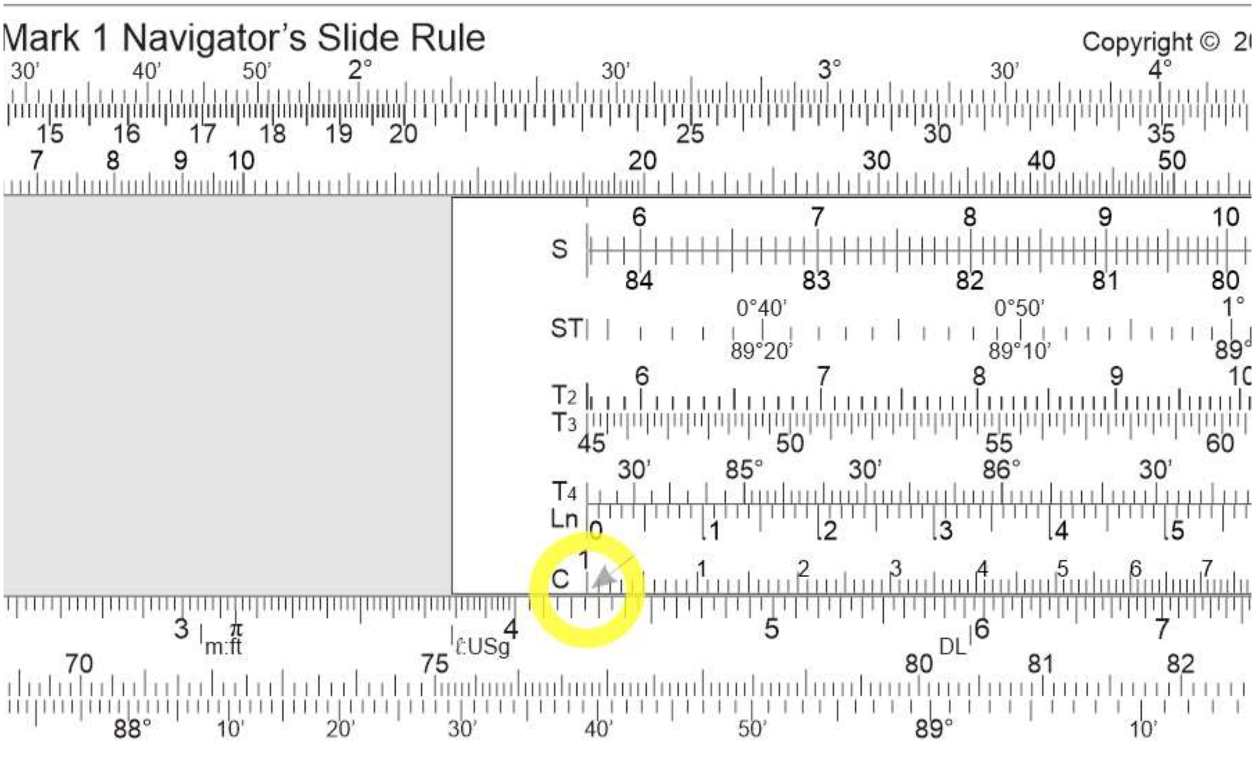


Sorting out the decimal place. You know that 3 × 300 = 900, so you know that your answer is in the neighborhood of 900. You know your answer is not 9.11. Nor is it in the neighborhood of 91.1.

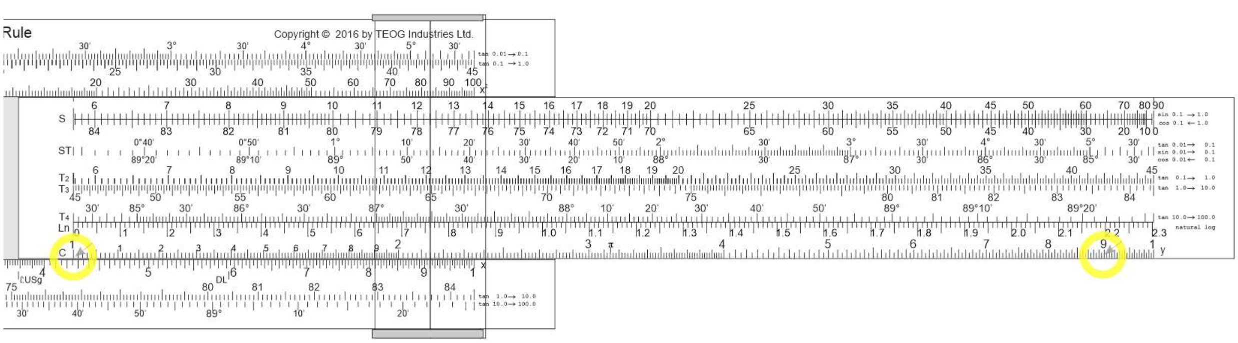
So when you see 911 on the slide rule, you know to position the decimal point at the end. Hence, 911.

Multiply 4,262 × 898

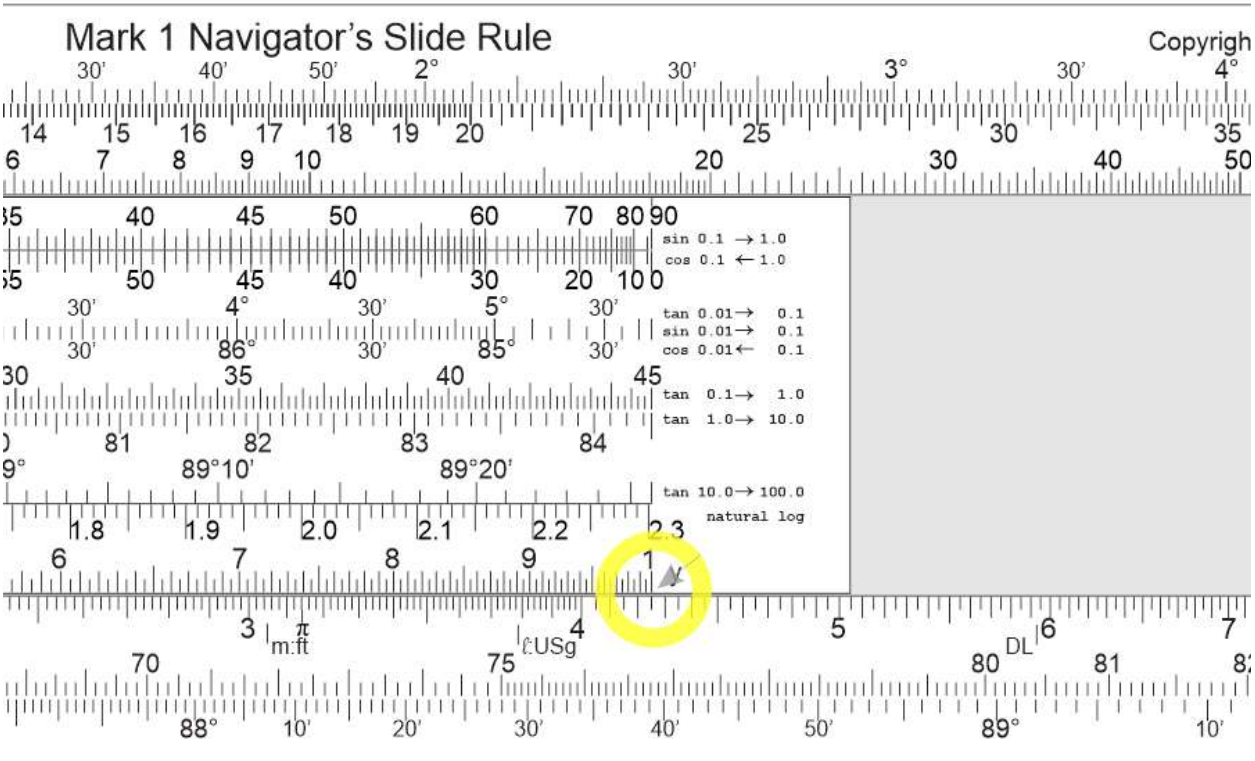
Position the left index over 426. The slide rule is only capable of 3 significant digits here, so you cannot come any closer to your number than 426.



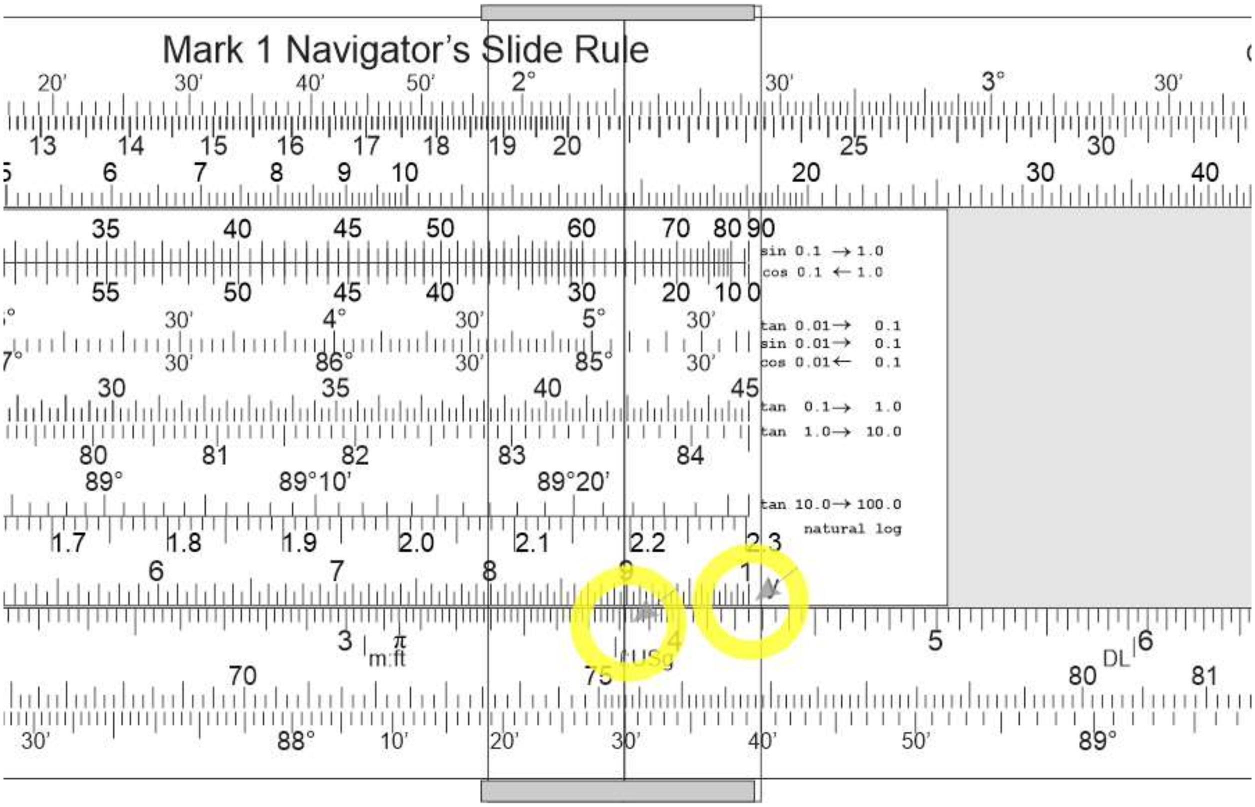
As you look at this, however, you see that you cannot slide the cursor over the top of 898. It extends beyond the right index on the D scale.



When this happens, use the other index on the C scale. That is, move the right index over the top of 426 on the D scale. Then slide the cursor over the top of 898 on the C scale.



The answer is 382.



Sorting out the decimal place. You dig back into your memories of learning to multiply big numbers in grade 5. In your head, you do this.

```
  4,000
×  1,000
  4,000,000
```

So you know your answer is somewhere in the neighborhood of 4 million. Since your slide rule says 382, you know your answer has to be 3,820,000.

When you work out this problem using the calculator on your smartphone, you get an answer of 3,827,276. But when you work this on your Mark 1, using 3 significant digits (a.k.a. slide rule accuracy), you get 3,830,000.

§35. *Chart Scales & Distance*

You charter a Beneteau 35 for that dream-sailing-vacation in the Caribbean. Because it is a charter boat, you are not quite sure what kind of navigation instruments you will find aboard. So you bring along your own parallel rule, dividers, binoculars and hand-bearing compass.

And then as you are going through airport security, your dividers are confiscated,[13] and when you get to your destination, you discover that indeed the on-board dividers are a total P.O.S. But no fear, you can calculate distances using the scale of inches on your Mark 1 Navigator's Slide Rule. Besides, this will give you an opportunity to practice basic multiplication on your slide rule.

Nautical Miles Per Inch = Reciprocal of Chart Scale ÷ 72,900

So then, if your chart scale is 1:50,000, you would do this:

nm/inch = 50,000 ÷ 72,900 = 0.686 nm/inch

Position the cursor over 50,000 on the D scale, and center 729000 from the C scale under the cursor. Read the result under the right index of the C scale.

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[13] ...because, you know, navigational instruments are so dangerous. My white-haired, 85 year old mother even had her cuticle scissors seized by an airport security person.

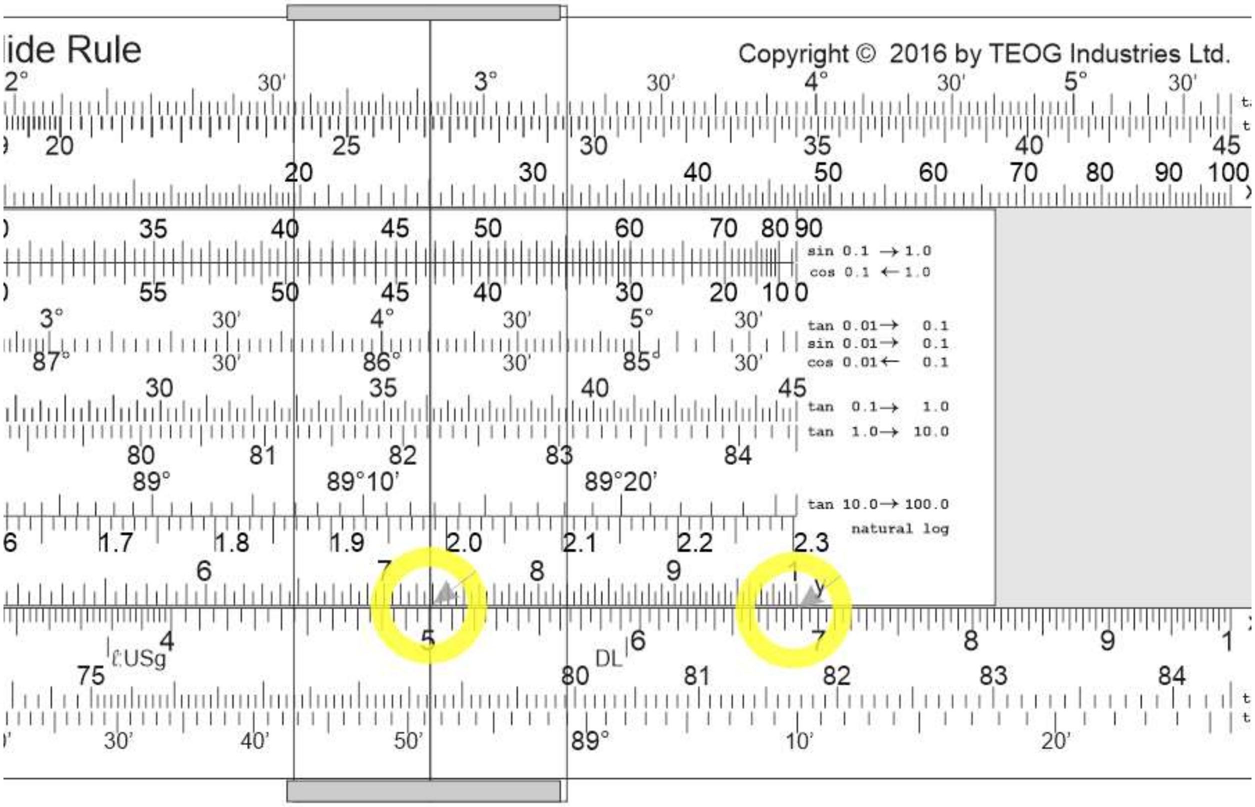
But I came to realize that this was only prudent, since my mom *could* have been a member of Al-Oughtrêd, a ruthless terrorist organization comprised of German grandmothers from Alberta and Norwegian grandmothers from Minnesota. They have been known to break through the armored door to the cockpit with no more than fingernail clippers, forcing the pilots to land in Somalia.

In 2014, Al-Oughtrêd began actively recruiting members from the ranks of the American Association of Retired People (AARP). So if you are a navigator over 50 with dividers in your carry-on luggage, you are liable to be profiled as a terrorist. If you have an AARP membership card, prepare for a strip search.

Oughtrêdis are so feared by jihadists that just the mention of the *name* "Al-Oughtrêd" can make members of Al-Qaeda and Al-Shabaab pee in their pants. And the possibility of actually *meeting* an Oughtrêdi is terrifying enough that members of ISIS will leave skid marks on their white robes.

In fact, it was a chance encounter with a French grandmother from Quebec at a Libyan training camp that led to an "incident" with an entire group of 87 ISIS members at once. To make matters worse, there was a CNN cameraman present and the images went viral.

It was this embarrassment that led to the current fashion of ISIS members wearing black garments. Much, much scarier if you can't see the brown stains.



Sorting out the decimal place. You know from looking at the slide rule that the significant digits of the answer are 686. But you still need to sort out whether the answer is 0.686, 6.86, or 68.6. Just looking at 50,000 ÷ 72,900, it is not instantly obvious how to position the decimal place.

It is in the nature of division that you get the same answer if you strike off digits equally from the end of the numbers concerned. Hence,

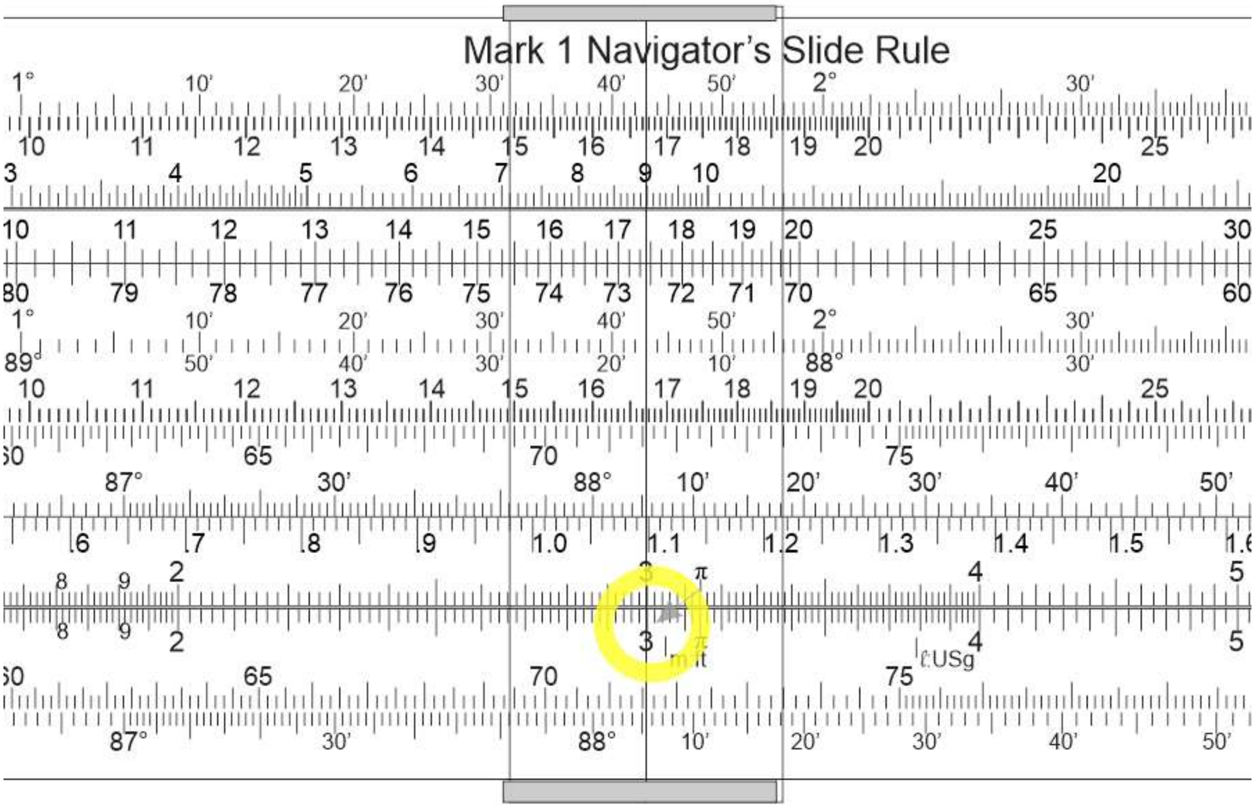
$\frac{50,000}{72,900}$ will yield the same answer as $\frac{50,0\not{0}\not{0}}{72,9\not{0}\not{0}}$ (or rather $\frac{500}{729}$) which will be the same answer as $\frac{5}{7.29}$.

You know that 5 ÷ 10 = 0.5, and that 5 ÷ 5 = 1, so the answer to 5 ÷ 7.29 will be somewhere between 0.5 and 1.0. Hence, you know that the decimal point should be 0.686 nm per inch.

So then, if you are trying to sort out the scale of a chart that is 1:500,000, then you can use the same approach to know that the answer will be 6.86 nm per inch.

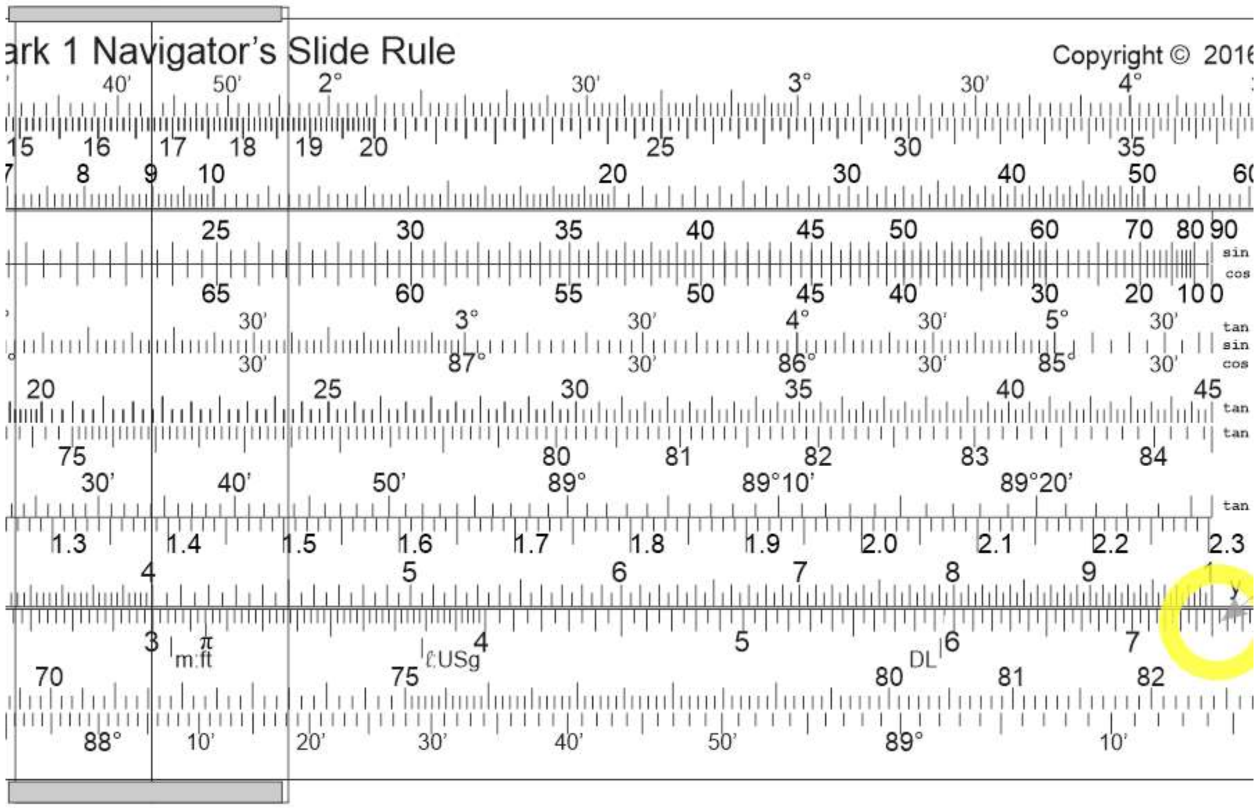
§36. Divide

To figure out $\frac{3}{4}$ on the slide rule, I find it easy if I verbalize it as “3 divided by 4”. In this case, I start by positioning the cursor over the top of the 3 on the D scale.



Then I position the 4 from the C scale over the top of the 3 on the D scale.

Then I slide my cursor over so it is over whichever index on the slide is still over a number on the body and read the answer on the D scale. In this case, the answer is 0.75.



Sorting out the decimal place. As with multiplication, you are going to need to do some rough mental math to sort out whether the answer is 750, 7.5, or 0.75.

You are already getting the picture that using a slide rule is going to force you to start doing some rough, order-of-magnitude calculations in your head. This makes using a slide rule harder than using an electronic calculator, to be sure.

But on the plus side, using a slide rule "cultivates in the user an intuition for numerical relationships and scale that people who have used only digital calculators often lack."[14]

The classic example of a failure to *think* about the numbers you are getting from your digital calculation device (be it a $5 hand-held calculator, or a multi-million-dollar computer) is the crash of the Mars Climate Orbiter in 1999.[15]

Had they had just one engineer with well-developed numerical intuition say, "Wait a second. These numbers don't look right", and then convinced others to take him seriously,

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14 https://en.wikipedia.org/wiki/Slide_rule, retrieved 03 March 2016.

15 The California scientists with NASA were all using metric units in their calculations, while the propulsion engineers in Colorado were all using US-customary (foot-pound) measurements in theirs. Nobody noticed that the navigational software in the spacecraft was confused until it was too late.

they could have saved the $327 million that went up in smoke as the spacecraft was incinerated in the Martian atmosphere.[16]

§37. *Fuel Consumption*

Diesel engines consume about 1 gallon of fuel per hour for every 18 horsepower used when the engine is run at 75% of its maximum RPM.

Another way of expressing this is to say that you can estimate the gallons of diesel fuel consumed by multiplying horsepower by 0.055.

Gallons per Hour = Horsepower x 0.055

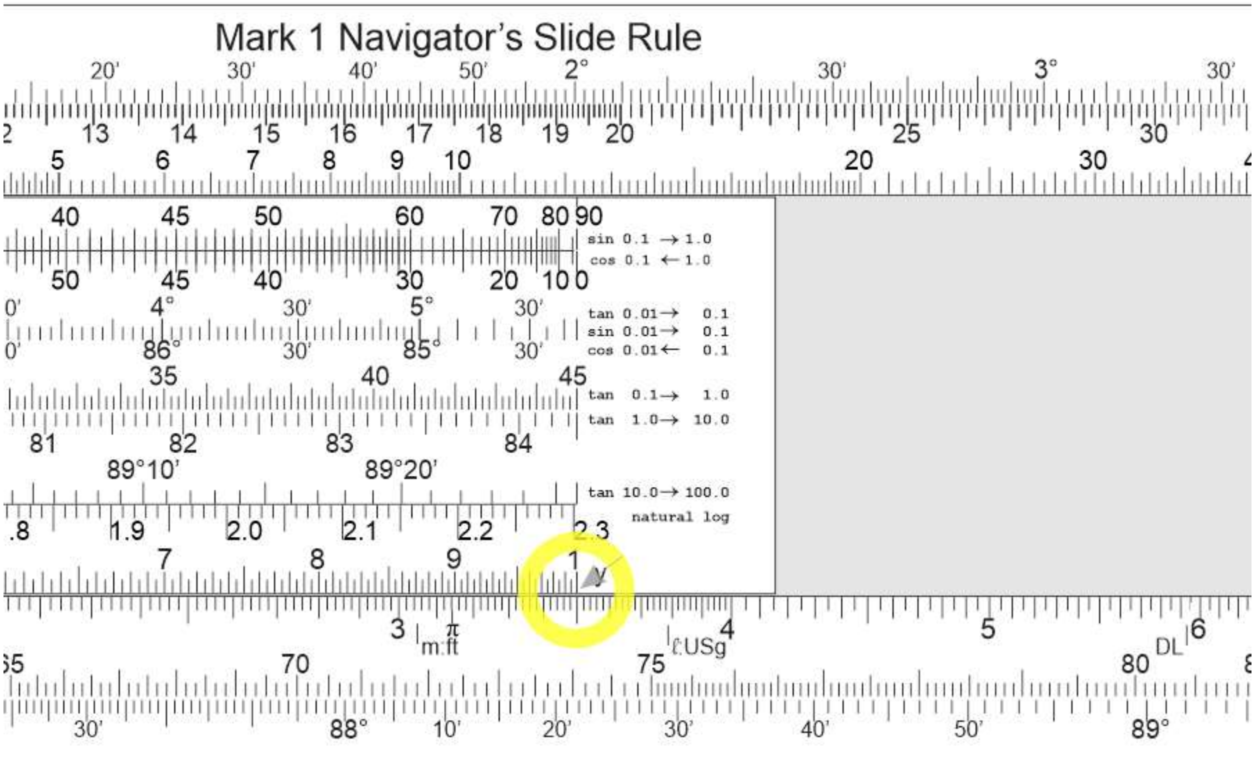
How much fuel will a 35 hp diesel consume while running 20 hours at 75% of maximum RPM?

GPH = 35 × 0.055

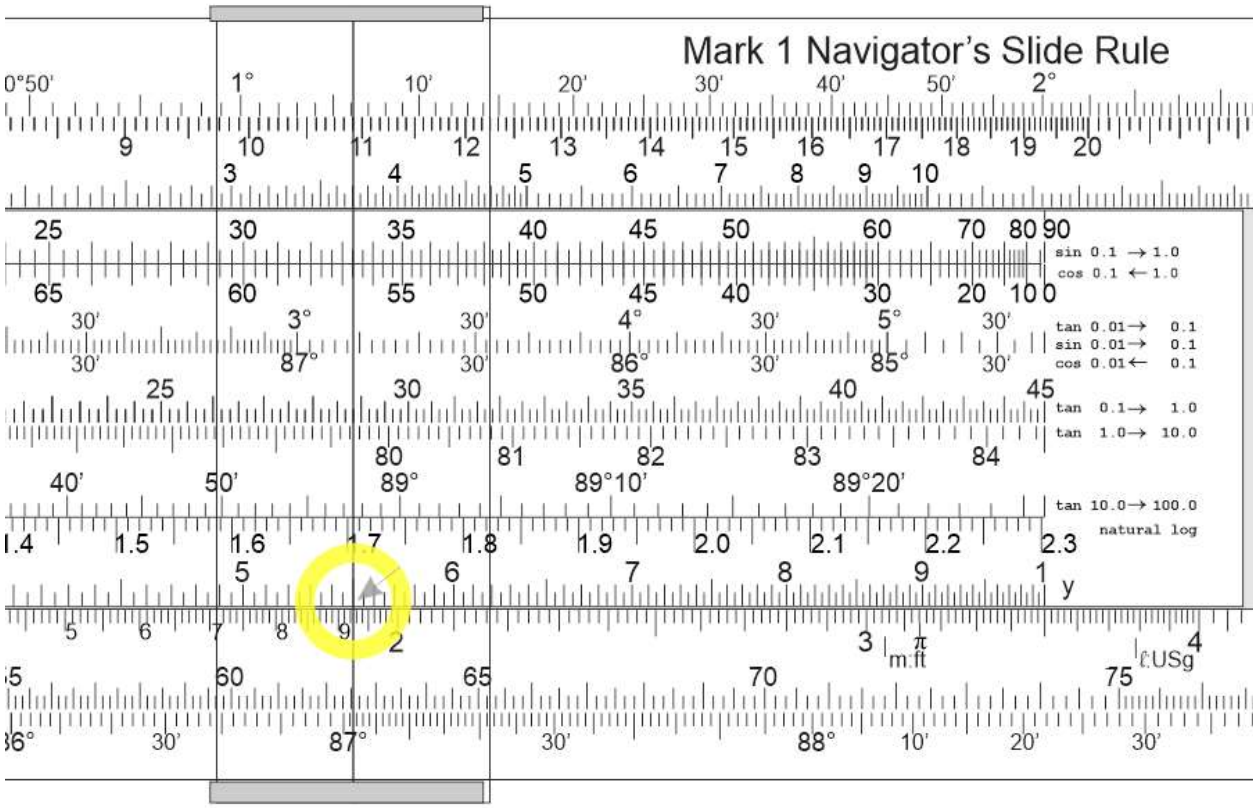
Gallons consumed = 1.926 GPH × 20 hours

Position the right index on the C scale over 35 on the D scale.

[16] It seems that most people today would rather get an answer that is wrong to 16 decimal places rather than get the right one to 3. A slide rule is, to quote Obi-Wan Kenobi, "An elegant tool for a more civilized age."

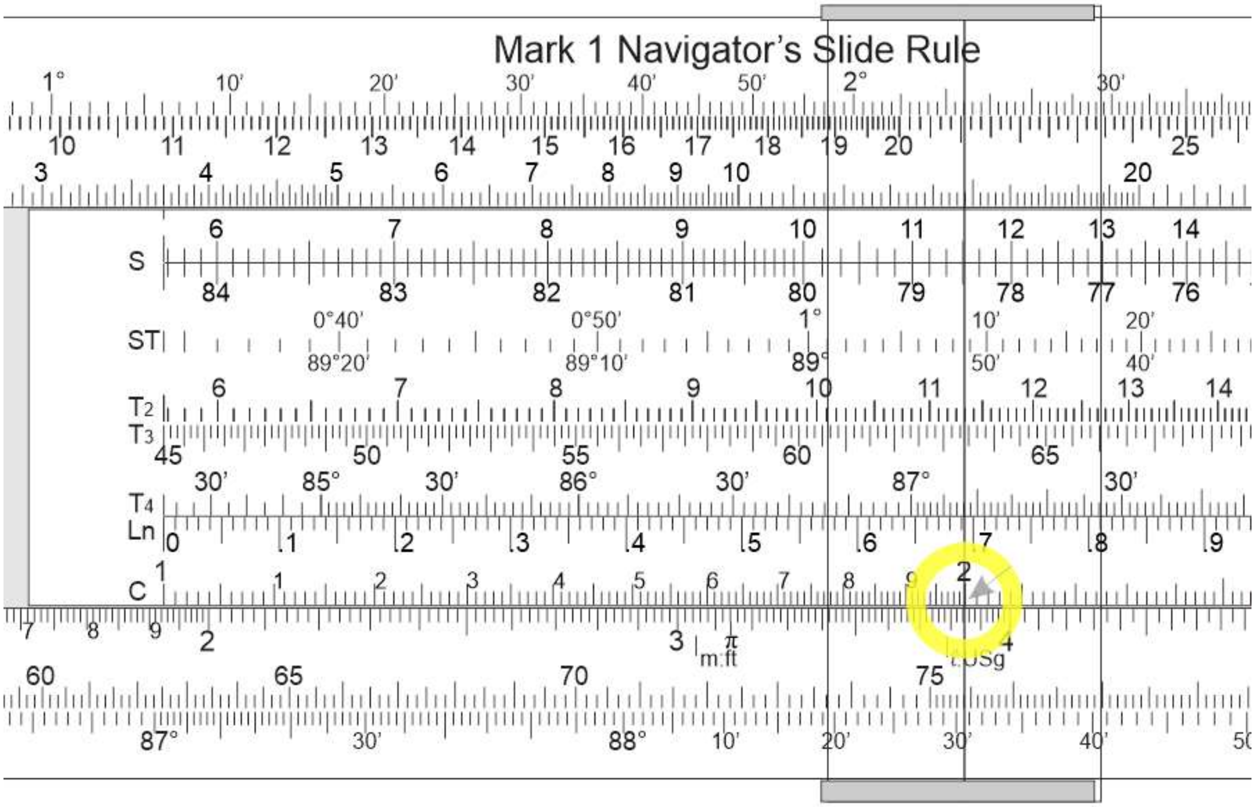


Slide the cursor over to 0.055 on the C scale. Read the answer of 1.926 gallons per hour on the D scale.



Sorting out the decimal place. You know that 0.055 is about ½ of 0.1. You can easily figure out that 35 × 0.1 is 3.5, and half of that is around 1.5 and change. So when you see an answer of 1926, you know that the answer is near 1.5, hence 1.926.

Finishing up: Move the left index over so it lines up underneath the cursor at 1.926.



You can read the answer as 38.6 gallons.

Sorting out the decimal place. You know that you boat is burning approximately 2 gallons per hour. If you run for 20 hours, you know you are going to burn in the neighborhood of 40 gallons. So when you see an answer of 386, you know immediately that the answer is neither 3.86, nor 386.0, but rather 38.6

There is another equation that plays a role in big-ship operations: $cS^3 = Cs^3$, to describe how much fuel you WOULD consume (C) at a new speed (S) if you know how much fuel you DO consume now (c) at your current speed (s).

That is to say, if you are consuming 38.6 gallons of diesel to get to your destination at 6 knots, how much would you consume if you kicked your speed up to 9 knots?

This is a useful equation with larger cargo vessels, who can increase their speed by 50%, e.g. from 12 knots to 18 knots, without approaching their hull speed (See *§40*).

It is pretty common for a 40 foot cruising sailboat to motor at 6 knots. But increasing that speed by 50% is usually out of the question, since such a vessel might have a hull speed of 8.2 knots. If you had a big enough engine, you COULD push that 40' yacht at 9 knots, but the amount of fuel you would consume would go far beyond what you would expect from $cS^3 = Cs^3$. Since this rule is designed primarily for small boat sailors, and there are no useful navigational equations that recreational sailors want which includes cubes, the K scale (which is how a slide rule handles cubes) has not been included.

Squares and Square Roots

§38. Distance to Visible Horizon

It feels like the earth is this vast, table-flat surface that stretches away from us into infinity. But in reality, the earth's surface is curving away from us at a dramatic rate. If your eye is 6 feet above water level, your actual horizon is only 2.87 nm away.

You may be seeing trees or buildings along the shoreline that are further away than 2.87 nm, of course…but you are not seeing the point where those trees/buildings actually meet the waterline. That point is over the horizon from you. The equations that allow you to calculate this are (in feet to the left, and meters to the right):

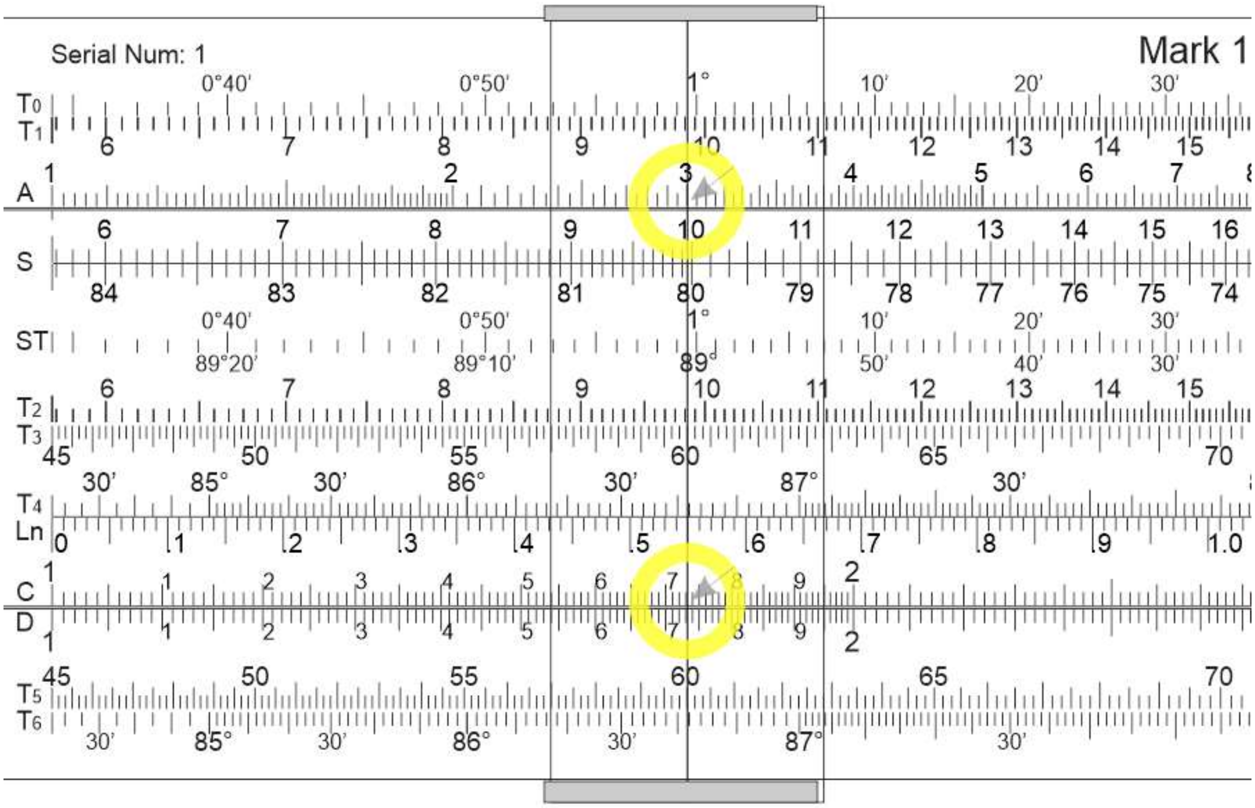
$D = 1.17 \times \sqrt{h}$ $D = 2.07 \times \sqrt{\text{height of eye in meters}}$

When I first got my sextant, in the early 1980s, I was living in Japan and sailing my 13' dinghy on Beppu Bay, on the NE coast of Kyushu Island.

I used the equation above to sort out that if my eye was 3' above the waterline, which is what it was when I sat in the bottom of the cockpit, the horizon was only 2.0 nm away. As long as I could get 2 miles offshore (easy to verify by the other navigational aids that were available) I could take sextant sights that would be as valid as if I were 1,000 miles offshore.

So, to calculate the distance to the horizon, with an eye height of 3', using a slide rule, you will use the A scale like this.

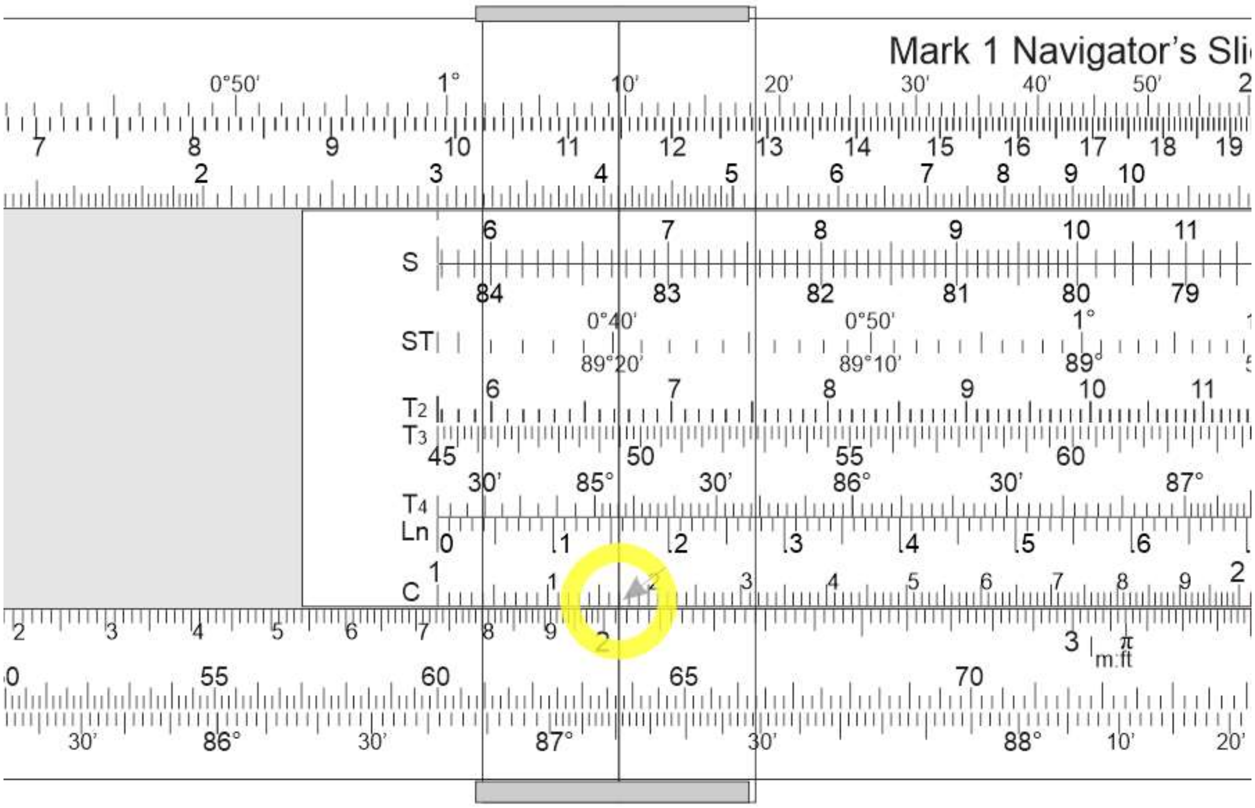
Position your cursor over 3 on the A scale. Once you have done this, you can read off the square root of 3 on the D scale on the body.



The square root of 3 is 1.732…though with slide rule accuracy, you may find that last digit is a little different.

One now needs to multiply this by 1.17. So move the left index so it is directly beneath the cursor.

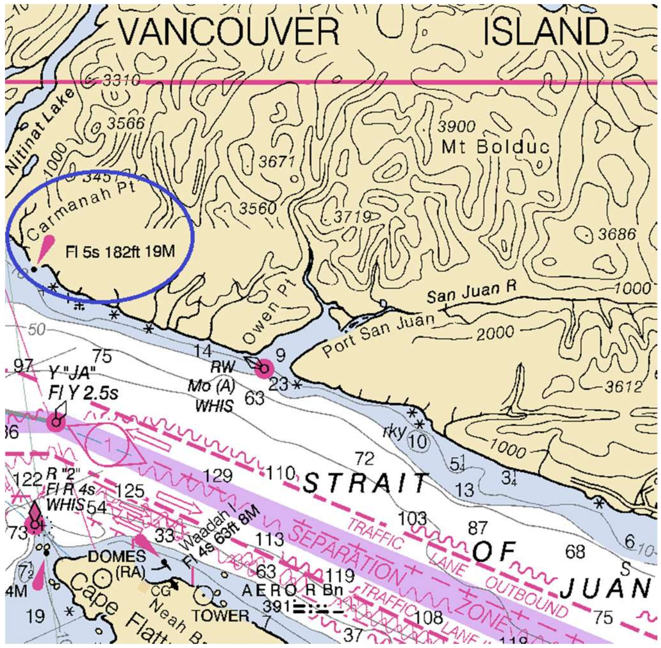
Now slide the cursor over to 1.17 on the C scale, and read the answer off the D scale: 2.025 nm.



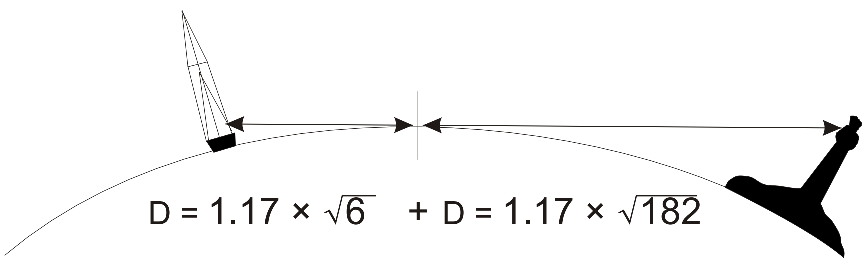
So the first use of this equation is in determining if you are far enough offshore to practice your sextant work exactly as if you were way offshore.

But there is another use for this equation. You are offshore at night, approaching the western entrance to the Strait of Juan de Fuca. You are on the lookout for the light at Carmanah Point. Your chart tells you that the light is 182 feet above the high water mark.[17]

[1717] This is the height of the light itself, not of the structure that houses the light. Also note, to be accurate, you must take the state of the tide into account to determine the height of the light above sea level at this moment.



If you figure where the horizon is as far as the light is concerned, and figure where the horizon is as far as your eye is concerned, you can know at what distance the Carmanah Point light will peek over the horizon.



Work this out, and you can know that the light will just peek over the horizon when you are 18.65 nm offshore. And a distance plus a bearing will give you a fix.

Calculating this on the slide rule will be a little faster if you think of it as…

$$D = (\sqrt{6} + \sqrt{182}) \times 1.17$$

The chart notates the nominal range of the light, i.e. the distance at which it is visible, based on the brightness of the light. As long as the nominal range is greater than the geographical range you have just calculated, then you will see the light peeking above the horizon at that calculated distance.

In the case of the light at Carmanah Point, the nominal range is 19 nm…so you are fine to see it when you come up on it at 18.65 nm.

§28.1 Decide Which Side of "A" Scale to Use

Look at the A scale, and you will see that it goes from 1 to 9, and then starts over at 1 again. So it is a scale that is split into two identical parts.

The tricky part of doing square roots is determining which side of the A scale to use. For example, the square root of 1.44 is 1.2 and the square root of 144 is 12 – both of which can be correctly read from the left side of the A scale. However, you need to read the square root of 14.4 from the right side of the scale to get the correct answer of ~3.8.

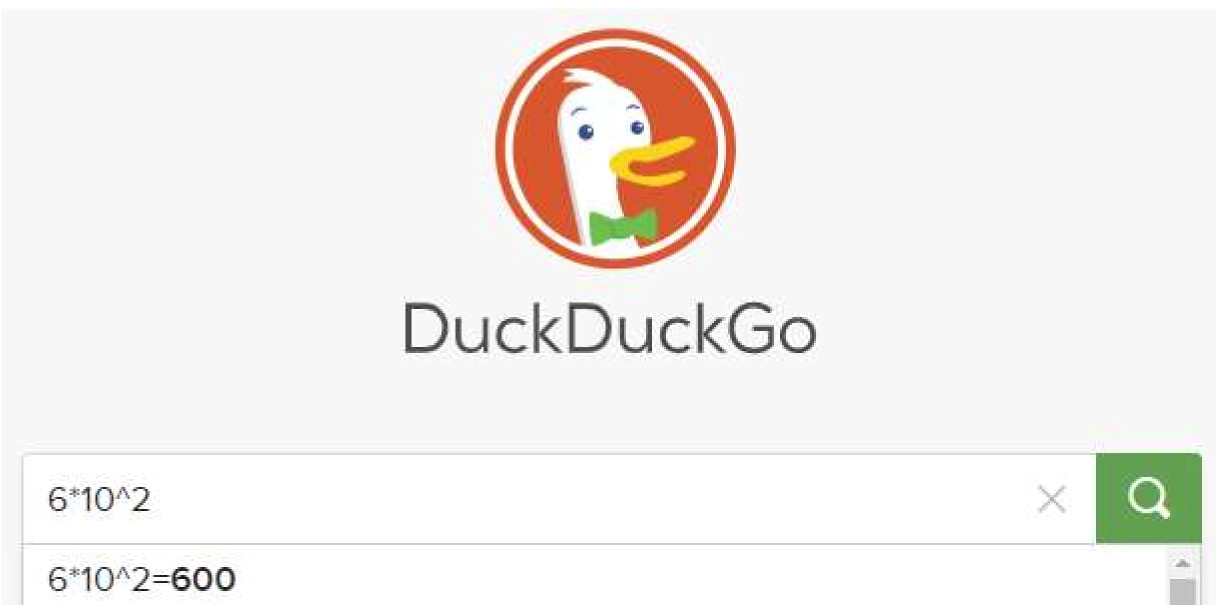
The simplest trick is to write the number in scientific notation (ie n.nnn x 10^exp) and use the left side for even powers of 10. For odd powers of ten, use the right side.[18]

6 = 6 x 10^0, so use the left side of the A Scale for that.

182 = 1.82 x 10^2, so it also uses the left side of the A Scale.

But if your height of eye was 12 feet (or 1.2 x 10^1), then you use the right side of the scale.

If your high school years are far behind you, and you are not comfortable with scientific notation, you can play around with it by going to www.DuckDuckGo.com and type an equation into the search field, and see the answer directly. Here is how you would type 6 x 10^2.



§39. Dip

The proper, accepted equation for calculating dip is:

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[18] https://hpmuseum.org/srinst.htm

$\text{Dip} = 0.97 \times \sqrt{h}$

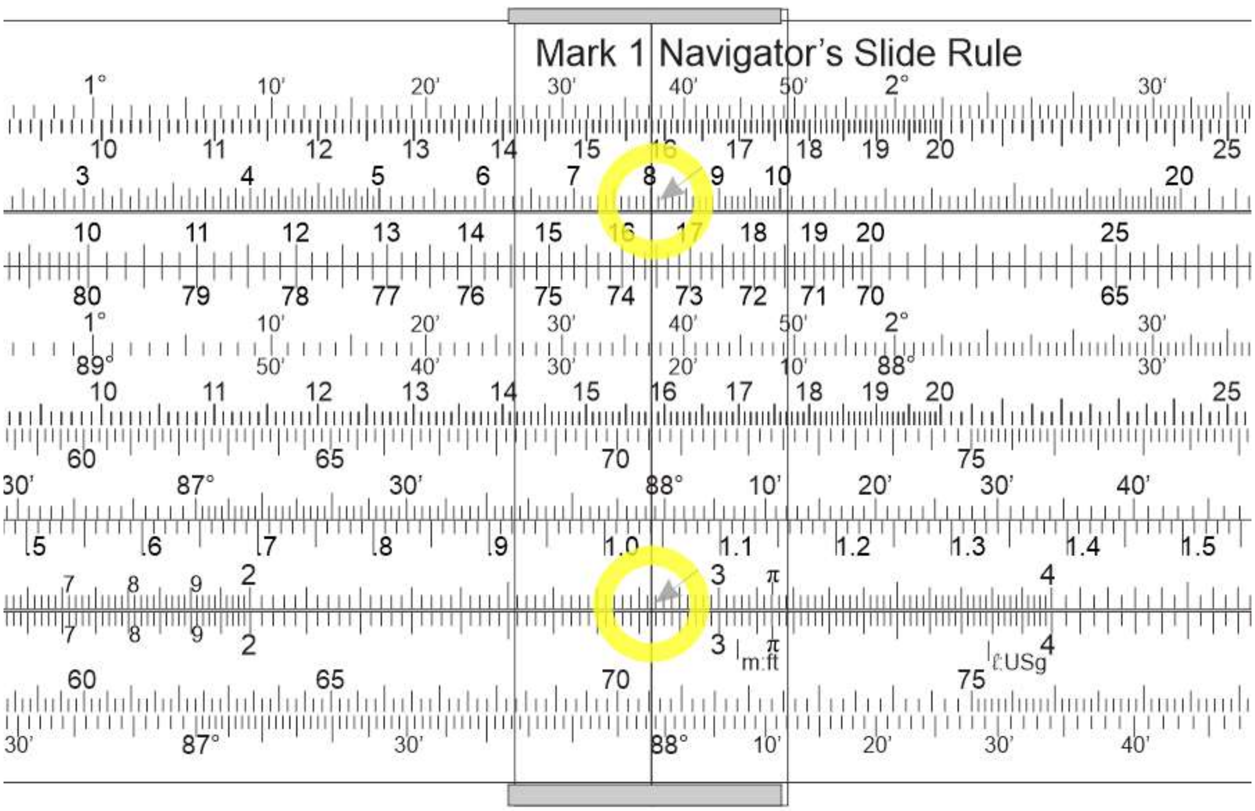
However, many navigators simply use:

$\text{Dip} = \sqrt{h}$

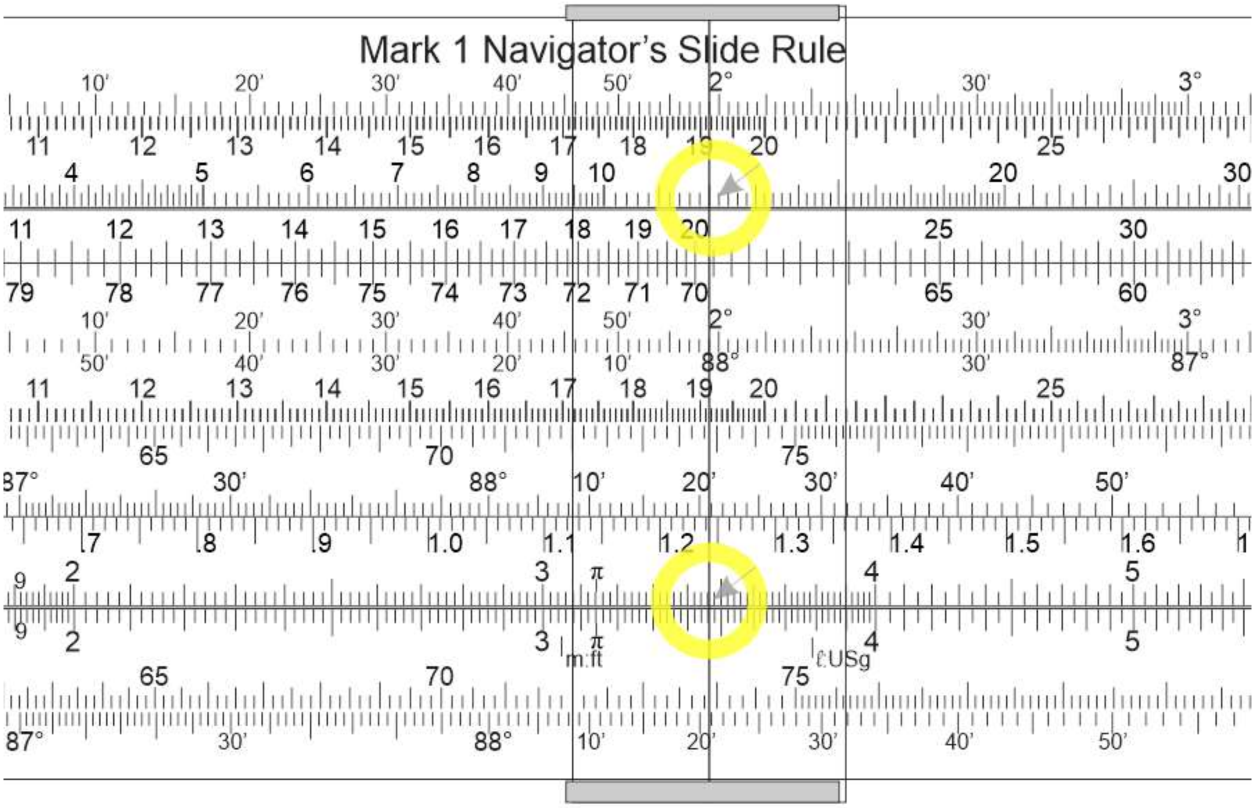
If you eye is 8’ above water level, the difference between the proper value in the first equation and the value from the second equation is less than 0.1’ of arc…navigationally insignificant.

Calculate the dip, in minutes of arc, where the height of the eye is 8 feet.

$\text{Dip} = \sqrt{8} = 2.8'$



The dip of 12 feet (using the right hand side of the A scale) is 3.5’.



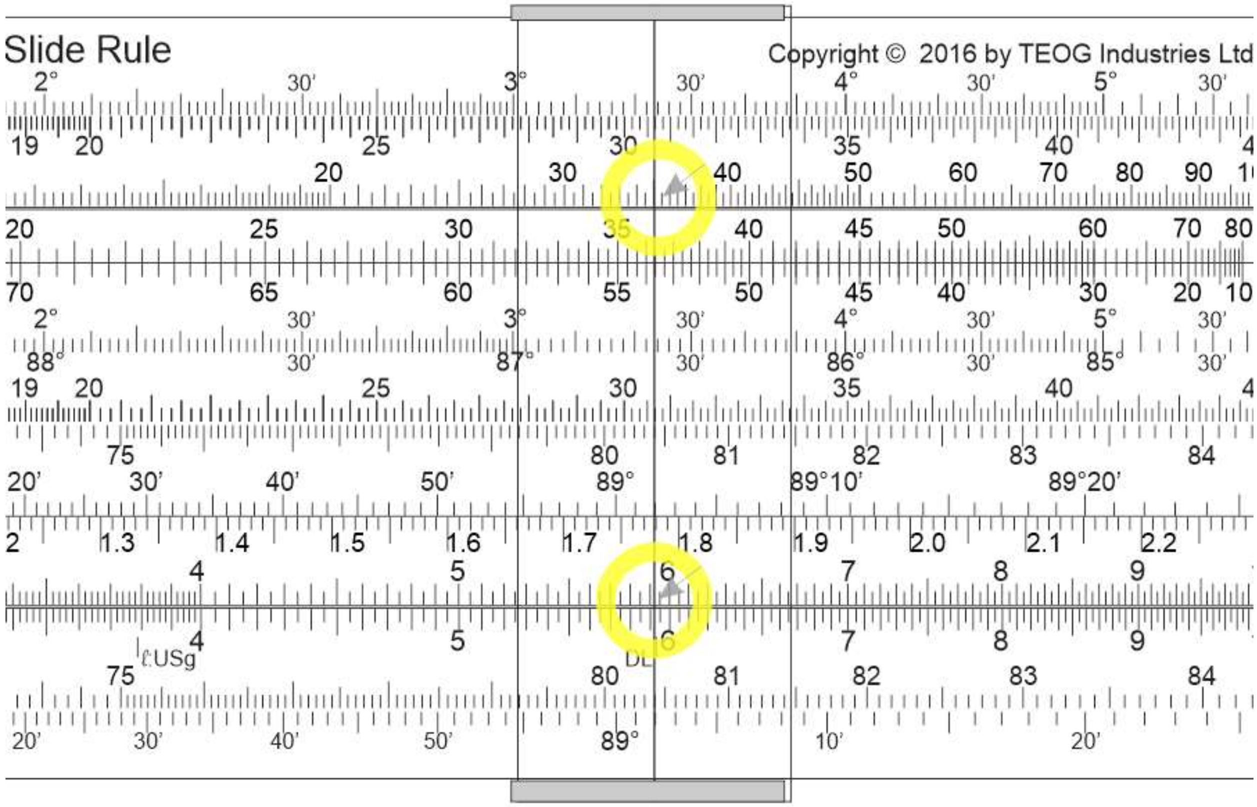
§40. Maximum Hull Speed

Another chance to practice your newfound skill in determining square roots will be to calculate the maximum hull speed of your vessel. This equation describes the approximate limit for hulls that are not planing. It is approximate, as there are differences based on hull shape…but there are some fundamental laws of physics that makes it more true than not.

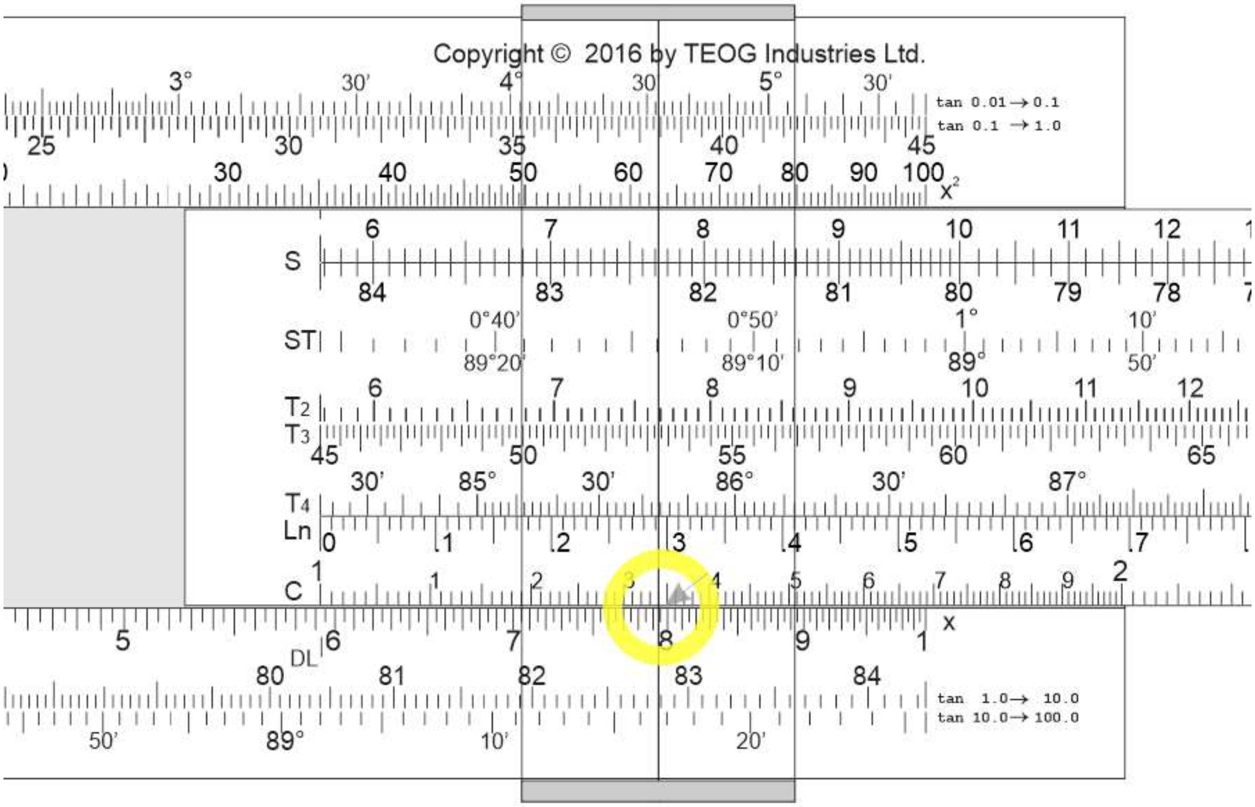
$S = 1.34 \times \sqrt{LWL}$

So if you have a vessel where the LWL (length at the water line) is 35 feet, you determine your hull speed as follows.

Position the cursor over the 35 on the right side of the A scale. Read the square root on the D scale: 5.92.



Then move the left index on the slide under the cursor, then move the slide over to 1.34 on the C scale, and read the answer on the D scale = 7.93 knots.



No matter how you trim your sails, or how hard you push your diesel, you are not really going to go any faster than this, unless you can get the hull up on a wave and plane.

As to why this equation works, it has to do with the physics of waves. http://tinyurl.com/jxynsw6[19] is an interesting article with a good illustrative picture.

A powerful vessel can often tow a smaller vessel at above its hull speed. When this happens, the stern of the smaller boat falls into the trough of the wave, the bow kicks up, and additional improvements in speed come only with a disproportionately larger increases in power for ever smaller increases in speed. http://tinyurl.com/jdmxy7f is a picture of a dinghy being towed at above hull speed.

Most cruising sailboats lack the power to go beyond hull speed, unless they are planing/surfing, or have exceptionally narrow-but-long hulls, such as some catamarans have.

Do note that LWL can vary as you are heeled over. So applying this equation may not be entirely straightforward. But LWL will never exceed LOA (length over all), nor matter how far the vessel is heeled.

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[19] Retrieved 11 March 2016.

Trigonometry

§41. Distance Off by Two Bearings and the Run Between

When reading navigational manuals, you will discover that there are a variety of “easy” rules to remember if you want the distance to an object by two bearings plus distance run:

- Four point fix
- Double the angle on the bow
- 7/10 rule
- 7/8 rule[20]

If you have a slide rule, however, then there is one quick and easy strategy that will work with ANY pair of bearings plus the run between.

Here is the basic scenario, where you are cruising toward a light or other navigational aid:

1. Note the log reading on your knotmeter at the moment you take the first bearing on the light.
2. Carefully maintain your course. (Speed can vary since you are using your electronic knotmeter’s log. If you don’t have such an instrument, then maintain a constant speed and calculate distance run as outlined in *§28*.)
3. After some time has passed, take another bearing on the object, and note the distance run from your log.
4. Subtract the smaller bearing from the larger, to get the difference.

You will be using this relationship, which looks scarier than it is:

$$\frac{\text{distance run}}{\sin(\text{difference between bearing 1 and bearing 2})} = \frac{\begin{array}{c}\text{distance from object}\\ \text{when bearing 1 was taken}\end{array}}{\sin(\text{bearing 1})} = \frac{\begin{array}{c}\text{distance from object}\\ \text{when bearing 2 was taken}\end{array}}{\sin(\text{bearing 2})}$$

Example:

Your vessel observes a light bearing 25°, and after running 9 miles, it bears 64°.

The difference between bearings is 64° - 25° = 39°

Position your cursor over 9 on the D scale. Move the slide until 39° on the S scale lies under the cursor.

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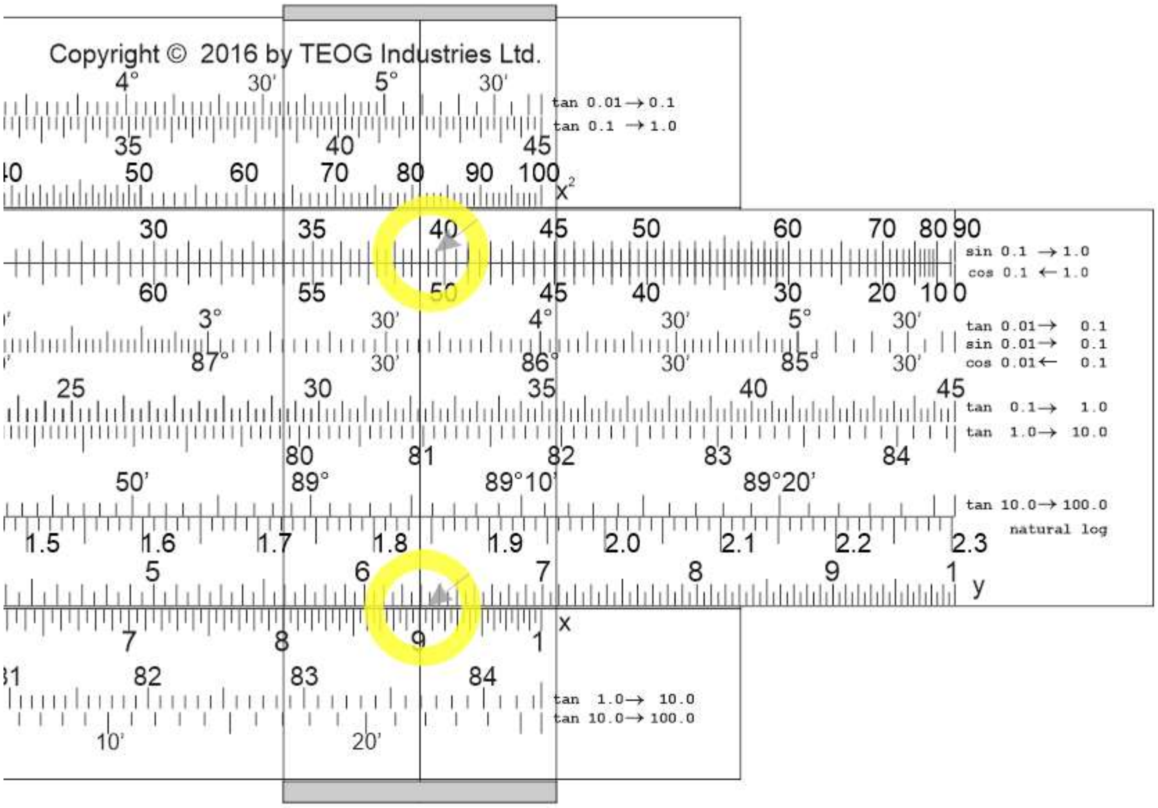
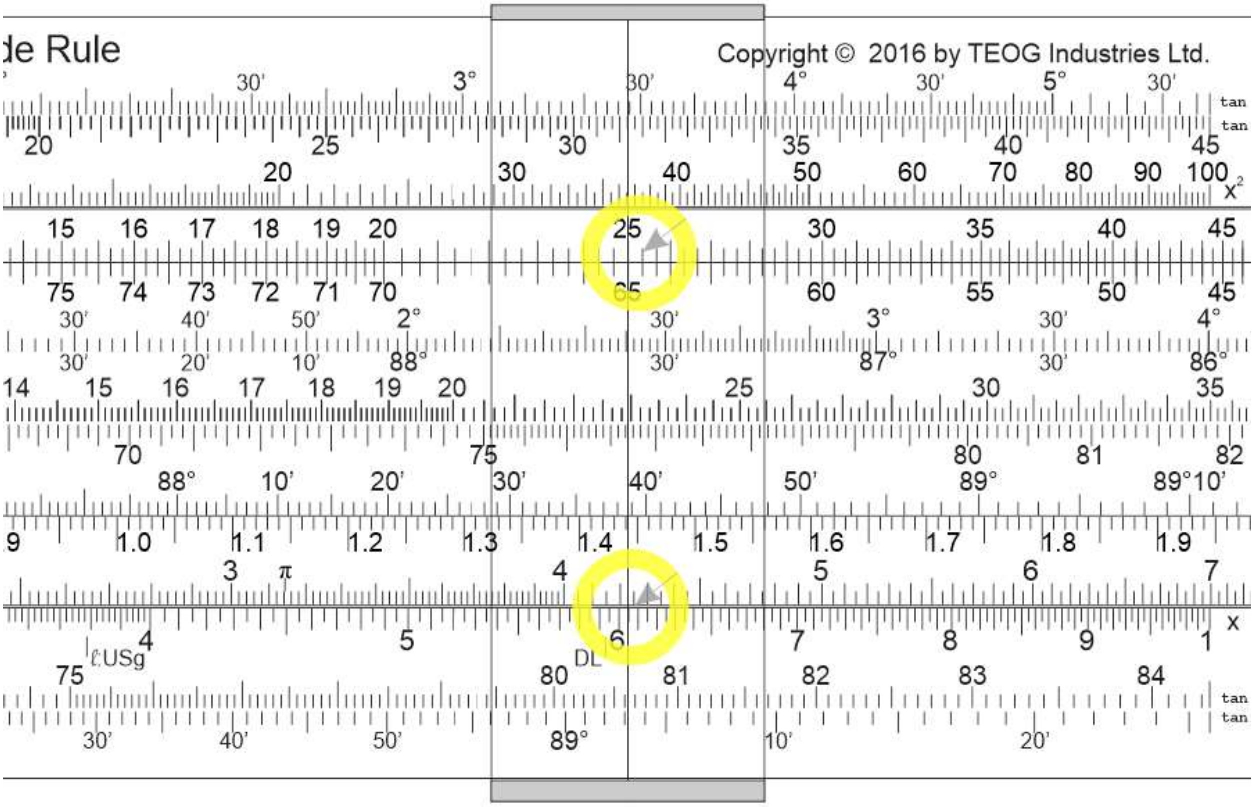
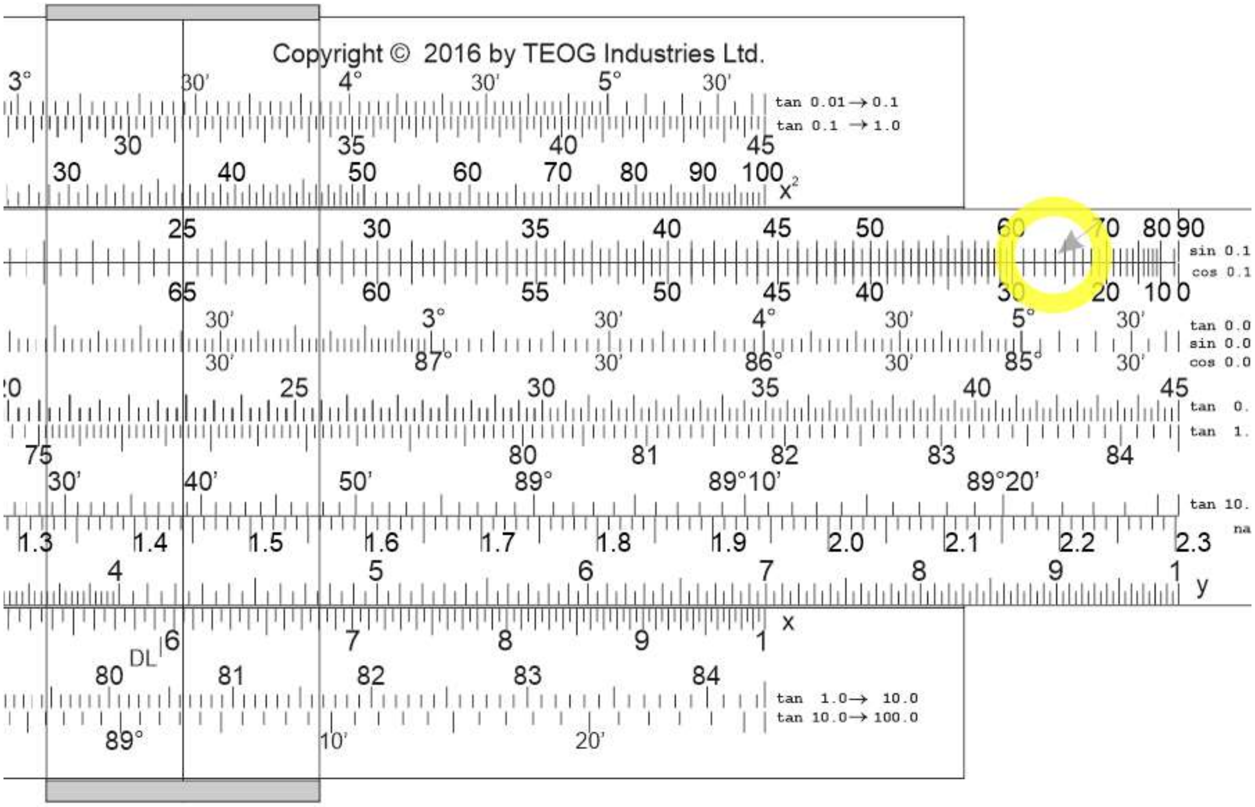
[20] Cf. Husick, Charles B., *Chapman Piloting & Seamanship* (New York: Hearst Books), 2009. Chapter 18.



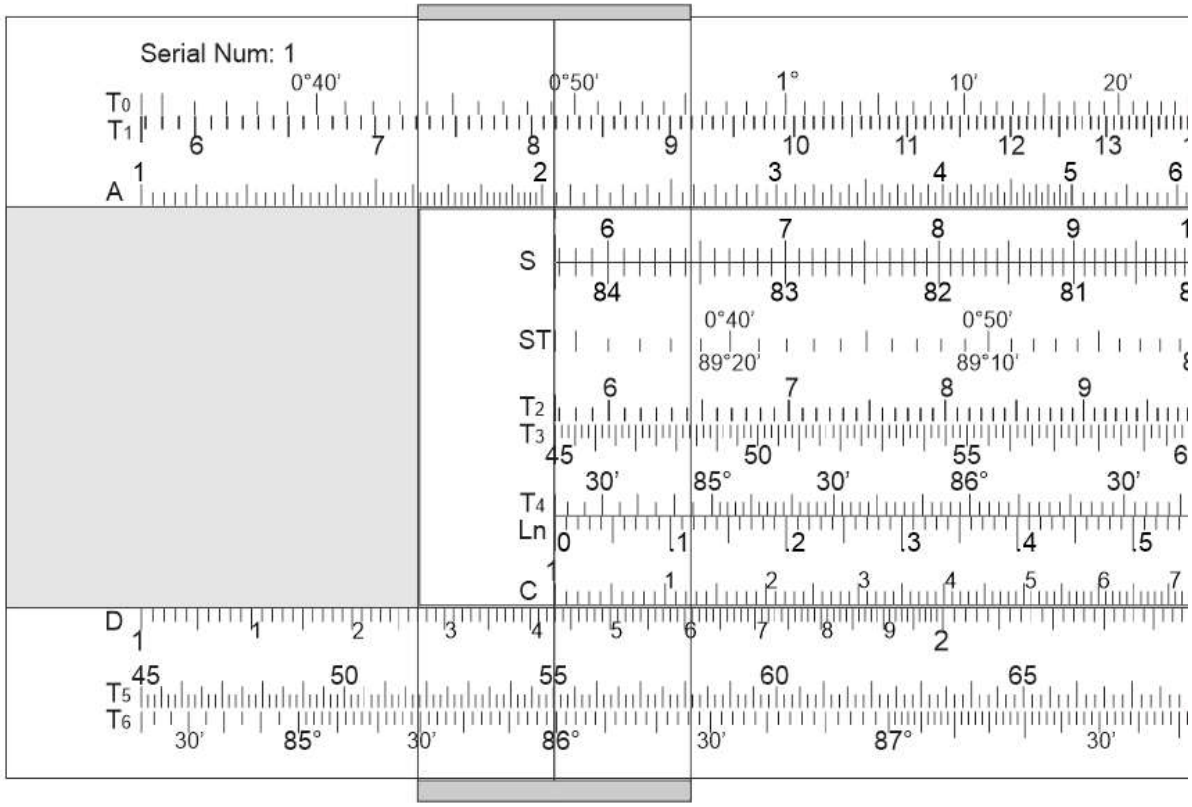
Now, slide the cursor over to the sine of the first bearing that you took: 25. Read off the D scale that when you took your first bearing, you were 6.05 nm from the light.



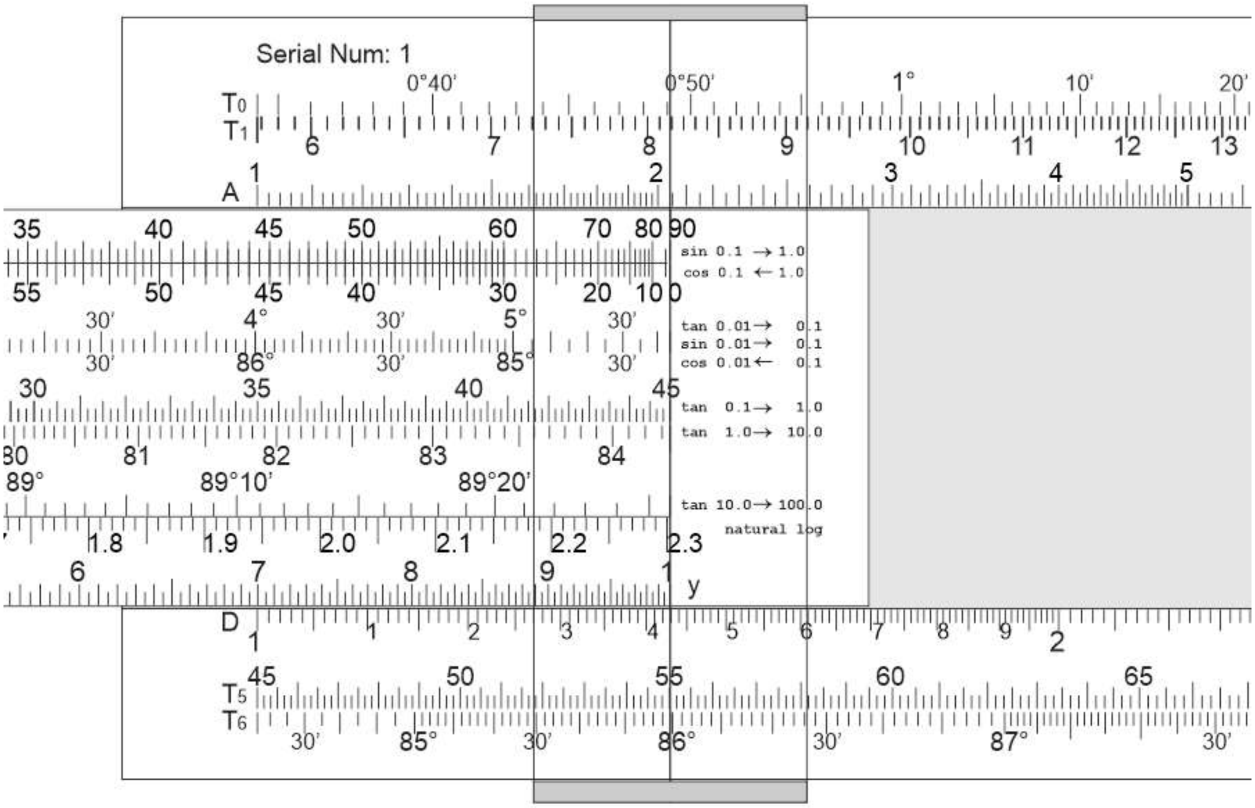
You cannot simply slide the cursor over the sine of 64°, the second bearing you took, as sin(64°) extends out beyond the right index on the D scale.



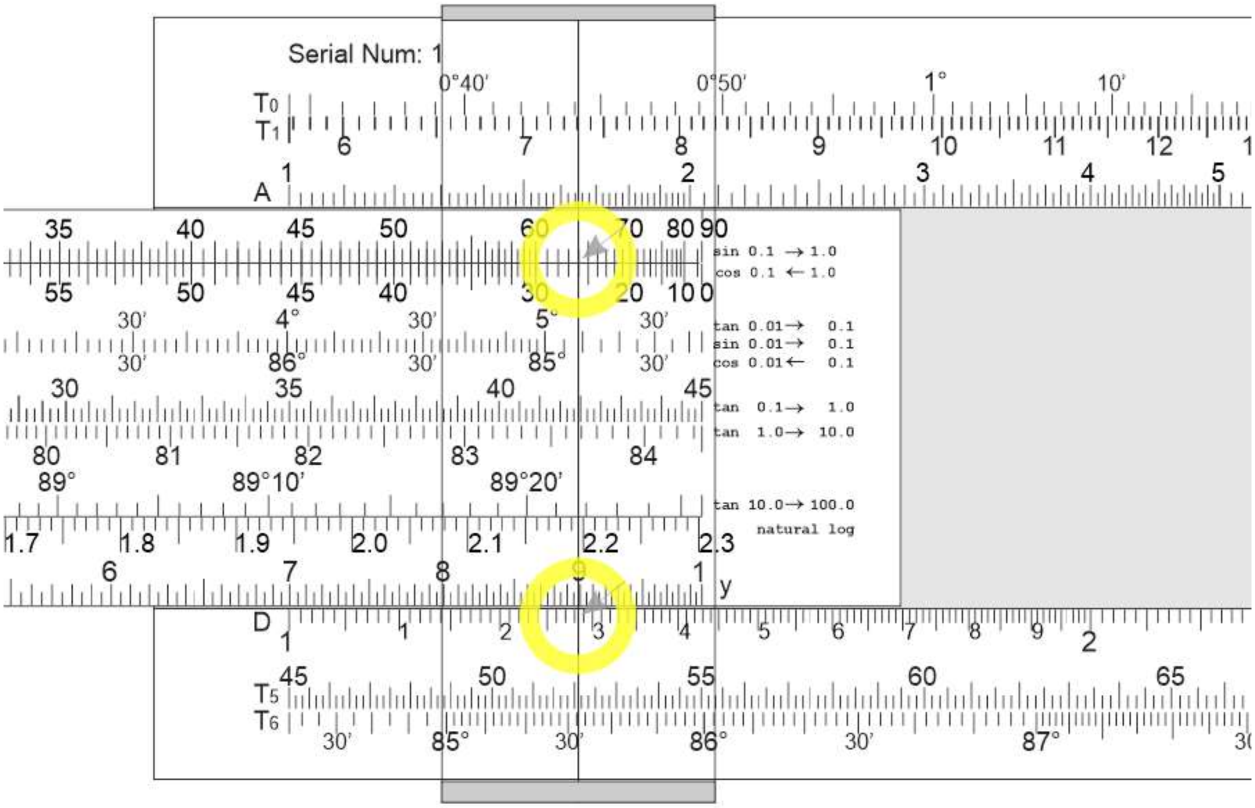
So slide the cursor over the left index on the slide.



Now move the right index of the slide until it is beneath the cursor.



Now, move the cursor until it is over 64°, the second bearing you took, and read the distance off at that point from the D scale: 12.85 nm.



With this data in hand, bearings and distances, you are ready to head down to the navigation station and plot your fixes and the course line you have been following[21]. You can observe in a heartbeat whether your current course will leave at a safe distance when the light is abeam[22], or if a course change is called for.[23]

§42. *Distance Off by Vertical Angle Measured Between Sea Horizon and Top of Object Beyond Sea Horizon*

While not necessarily a slide rule technique (e.g. if the vertical angle is less than 0° 34.4' – the minimum that the Mark 1 can manage in tangent calculations), this is such an elegant bit of navigational mathematics, I thought I should include it here.

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[21] This is one of the calculations you can make with a slide rule, with almost no practice at all, more quickly than you can with an electronic calculator. The technique is so easy and fast, it may well become a favorite in your navigational toolbox.

[22] It is often the case that lights are on points of land that have a region of shoals extending some distance offshore from that point.

[23] While the quickest method to solve this problem is with your pencil and straightedge, if you are utterly addicted to navigational math, you can calculate your distance off – if you continue your current course – with this equation:

Distance-off-when-abeam = Distance-off-at-Bearing-Point-2 × cos(90° minus bearing-off-the-bow-at-point-2)

Constants for refraction and curvature of earth = 0.0002419 and 0.7349
Height = difference in feet between height of object and eye of observer
Distance is in nautical miles

$$\text{Distance} = \sqrt{[\tan(\text{angle}) \div 0.002419]^2 + [\text{height} \div 0.7349]} - [\tan(\text{angle}) \div 0.0002419]$$

§282.1 By Solar Powered Calculator

An illustration of this would be to assume that you use a magnetic hand-bearing compass to determine that Cape St. Vincent, the southwest corner of Portugal, bears 58° M. The variation is 2° W, so the true bearing is 56°.

You know you lie somewhere along this line of position, but you don't know where. There are no other convenient points to sight with your hand bearing compass.

Your height of eye is 6 feet, and you know by DR that you are more than 3 nm offshore. So you know that the base of the cliff is below your visible horizon[24]...which makes this equation useful to you.

Height of Tower =	282 feet
Height of Tide =	1 foot
Height of Eye =	6 feet
Difference =	**275 feet**

You look through your sextant, and you see that from the top of the lighthouse to the horizon is 0° 07.7'. So then:

$$\text{Distance} = \sqrt{[\tan(\text{angle}) \div 0.002419]^2 + [\text{height} \div 0.7349]} - [\tan(\text{angle}) \div 0.0002419]$$

$$\text{Distance} = \sqrt{[\tan(7.7') \div 0.0002419]^2 + [275 \div 0.7349)]} - [\tan(7.7') \div 0.0002419]$$

$$= \sqrt{9.25937557834^2 + 374.200571506} - 9.25937557834$$

$$= 21.4461326958 - 9.25937557834$$

$$= 12.2 \text{ nm}$$

In spite of my love of slide-rule navigational math, it is worthwhile having a solar powered calculator aboard your boat. The Casio fx-260 is probably the best combination of cheap ($10 CDN at Staples[25]) and capable (it does trig and is seaworthy[26]) that can be had in an electronic calculator right now. Unlike many scientific calculators, it is capable of accepting input in degrees-and-minutes, rather than just in decimal-degrees.

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[24] If your HoE = 6 feet, the horizon is 2.8 nm away.
[25] https://www.staples.ca/products/2520833-en-casio-fx260-solarii-scientific-calculator
[26] http://fer3.com/arc/m2.aspx/Calculator-dunking-FrankReed-jun-2015-g31753

§282.2 By Slide Rule

Of course, if you are measuring the peak of a Caribbean volcanic peak, you may have angles great enough that you can use this equation with your slide rule.

I worked a sample calculation where:

Height = 1,200 feet
Angle = 1° 00.0’

By electronic calculator, I came up with a distance-off of 10.12 nm. By slide rule, I came up with a distance-off of 10.7 nm.

If you are sailing in the Pacific, and are approaching Tahiti, whose highest point is 7,352 feet high, you will have plenty of opportunity to use your slide rule in distance-off calculations to estimate when you will reach port.

§282.3 By Bowditch

One other way of addressing this particular navigational problem is by going to Bowditch (2019), Volume 2, page 129 (available for free download[27]).

TABLE 15
Distance by Vertical Angle
Measured Between Sea Horizon and Top of Object Beyond Sea Horizon

Angle	Difference in feet between height of object and height of eye of observer											Angle
	100	120	140	160	180	200	250	300	350	400	450	
° ′	Miles	Miles	Miles	Miles	Miles	Miles	Miles	Miles	Miles	Miles	Miles	° ′
0 00	11.7	12.8	13.8	14.8	15.7	16.5	18.4	20.2	21.8	23.3	24.7	0 00
0 01	10.5	11.6	12.7	13.6	14.5	15.3	17.3	19.0	20.7	22.2	23.6	0 01
0 02	9.5	10.6	11.6	12.5	13.4	14.3	16.2	17.9	19.6	21.0	22.5	0 02
0 03	8.6	9.7	10.7	11.6	12.5	13.3	15.2	16.9	18.5	20.0	21.4	0 03
0 04	7.8	8.8	9.8	10.7	11.6	12.4	14.3	16.0	17.5	19.0	20.4	0 04
0 05	7.1	8.1	9.0	9.9	10.8	11.5	13.4	15.1	16.6	18.1	19.5	0 05
0 06	6.5	7.5	8.4	9.2	10.0	10.8	12.6	14.2	15.8	17.2	18.6	0 06
0 07	6.0	6.9	7.7	8.6	9.4	10.1	11.9	13.5	15.0	16.4	17.7	0 07
0 08	5.5	6.4	7.2	8.0	8.8	9.5	11.2	12.8	14.2	15.6	16.9	0 08
0 09	5.1	5.9	6.7	7.5	8.2	8.9	10.6	12.1	13.5	14.9	16.2	0 09
0 10	4.7	5.5	6.3	7.0	7.7	8.4	10.0	11.5	12.9	14.2	15.5	0 10

In using this table for a height of 275 feet, and a sextant reading of 0° 07.7’, I needed to extrapolate.

27 https://msi.nga.mil/NGAPortal/MSI.portal?_nfpb=true&_pageLabel=msi_portal_page_62&pubCode=0002

	250 feet	275 feet	300 feet
7.0′	11.9 nm	12.7 nm	13.5 nm
7.7′		12.2 nm	
8.0′	11.2 nm	12.0 nm	12.8 nm

§43. Scope of a Given Universal Plotting Sheet

When you configure a VP-OS plotting sheet, you are, according to Bowditch [28] and Dutton,[29] essentially making a Mercator chart for a small area. It is accurate for a particular range of latitude. But if you move outside that range, then you need to create a new sheet and re-establish the width of a degree of longitude.

If you want your plotting sheet to be accurate to within 1%,[30] then:

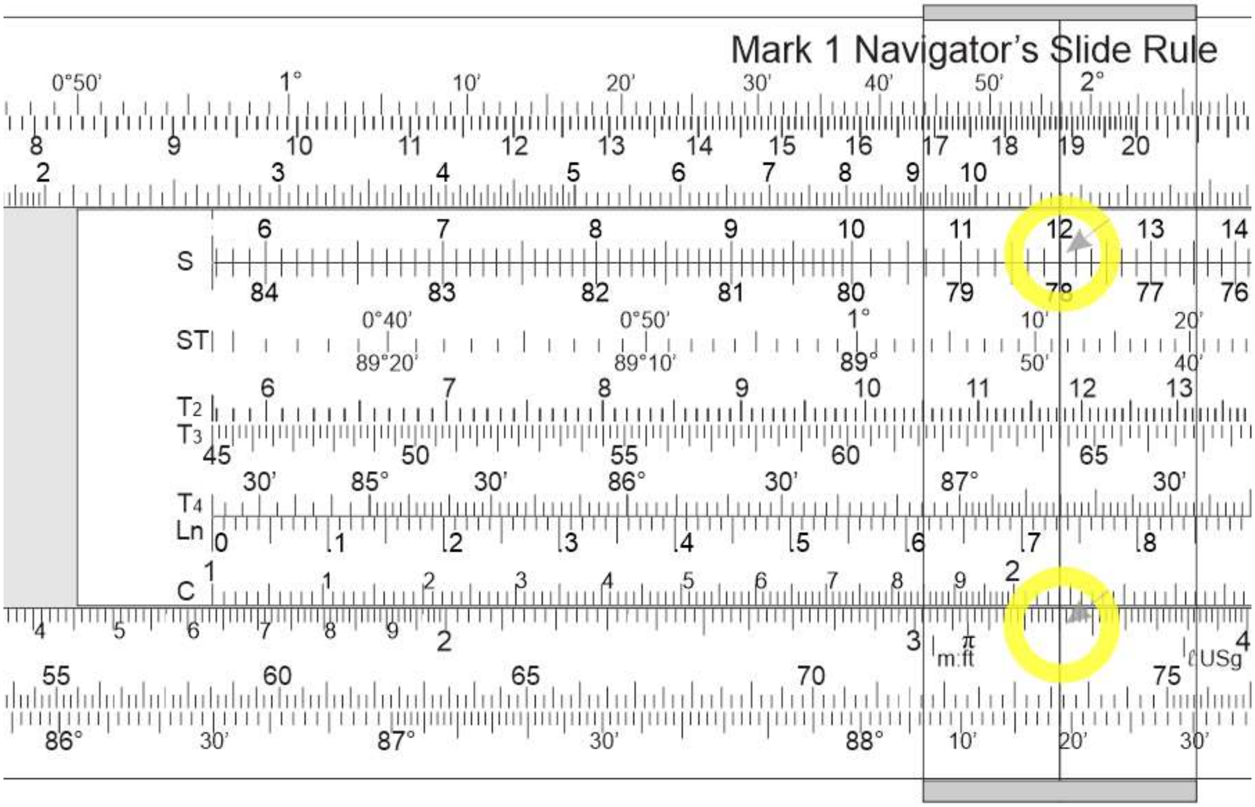
Usable latitude range in minutes = 34 ÷ sin(Latitude)

Imagine you are sailing at 12° latitude.

Step 1: Position the cursor over 34 on the D scale.

Step 2: Move the slide until 12° from the top half of the S scale is underneath the cursor.

[28] Bowditch, §2603.
[29] Dutton, 12th edition, §317.
[30] http://tinyurl.com/hd6t7zj (retrieved 04 March 2016).



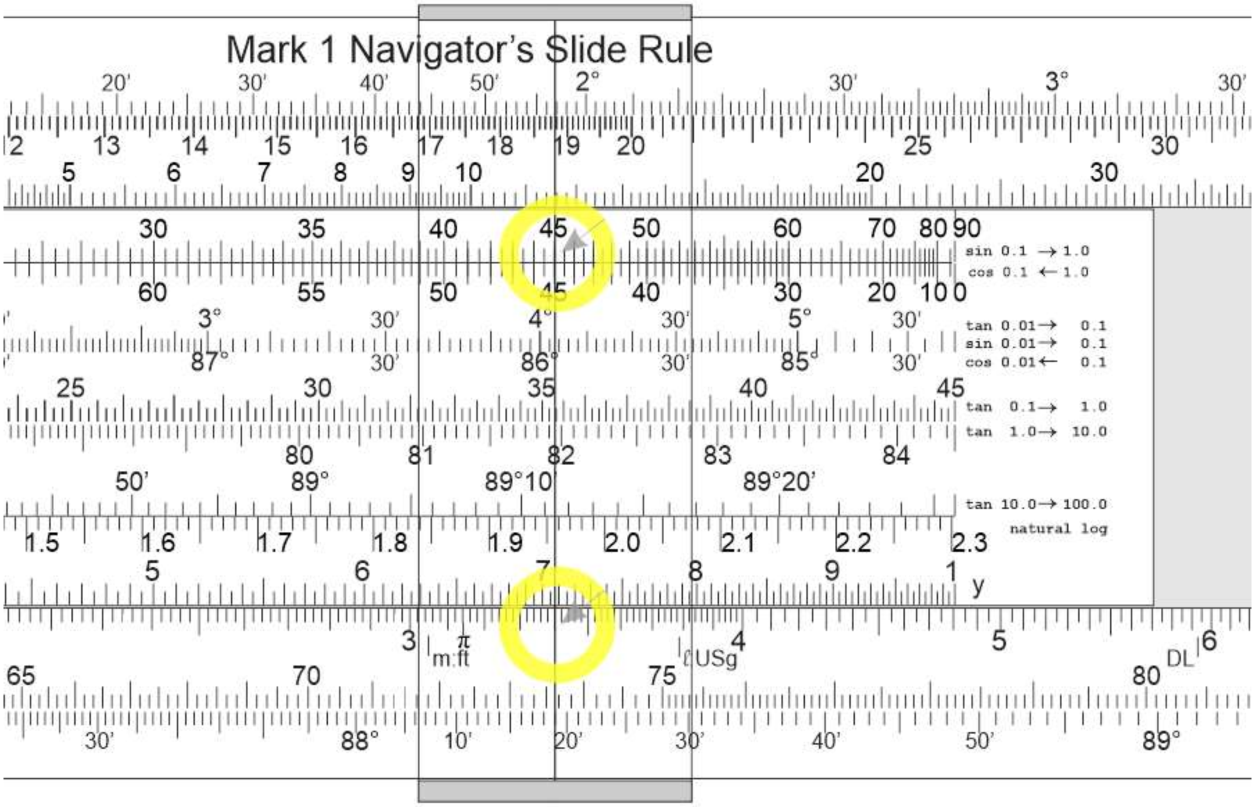
Step 3: Read the answer on the D scale, immediately under the left index of the C scale, i.e. 163'.

So a plotting sheet constructed for 12° latitude will have acceptable accuracy for positions ± 163' of latitude from 12°, or ± 2° 43', or within a latitude range from 9° 17' to 14° 43'.

If you are sailing at 45° of latitude, then:

Step 1: Position the cursor over 34 on the D scale.

Step 2: Move the slide until 45° from the top half of the S scale is underneath the cursor.



Step 3: Read the answer on the D scale, immediately under the left index of the C scale, i.e. 48'.

So a plotting sheet constructed with a central latitude of 45° will have acceptable accuracy for positions ± 48' of that latitude, or within a latitude range from 45° 48' to 44° 12'. Hence, you will want to construct a new plotting sheet for central latitudes of 44° or 46°.

§44. Sight Reduction/Great Circle Route Calculations

The basic method for solving a spherical triangle was worked out in ancient Greece. Different variations of this solution have been formulated to try and optimize for different tools and scenarios.

The solution that gives the best results when doing calculations with a slide rule was developed in 1920 by Captain L.G. Bygrave[31] of the Royal Air Force. Plotting a great

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[31] Captain Bygrave's original slide rule was actually cylindrical rather than straight. The advantage of this design was that the scales, if unwound, would be many feet long. It gave much better accuracy than a 10" straight slide rule. You can see a picture of one at https://en.wikipedia.org/wiki/Bygrave_slide_rule.

Gary LaPook has done extensive research and writing about the Bygrave, and the worksheet in this section for solving the navigational triangle, is entirely derived from Gary's website at http://tinyurl.com/BygraveSlideRule.

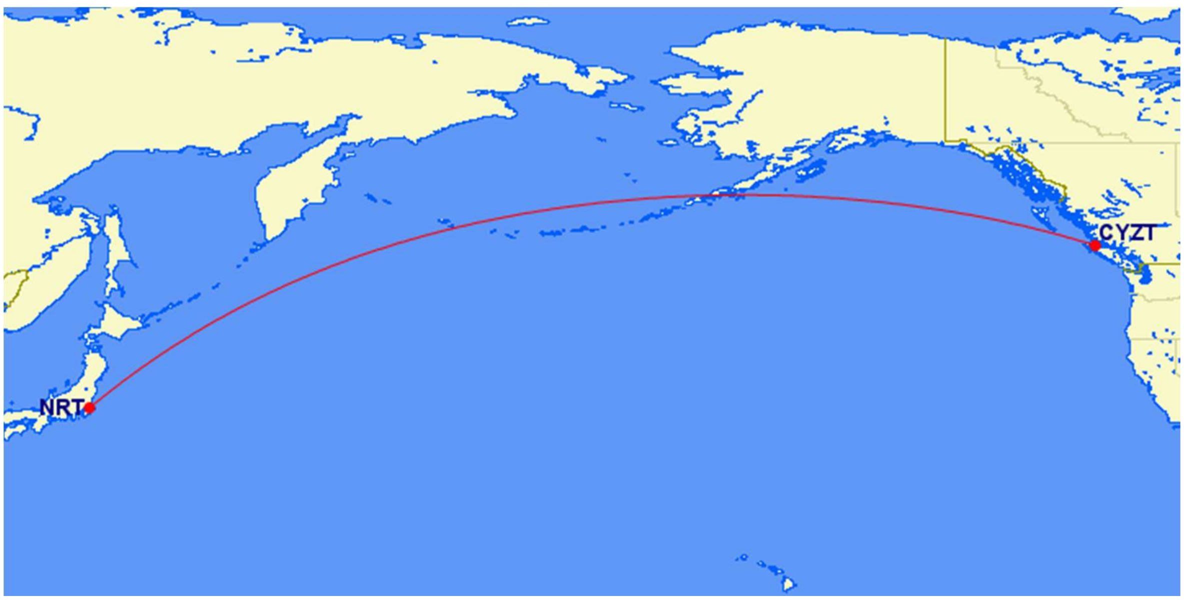
circle route is precisely the same calculation as you would use for sight reduction of a celestial sight.

On balance, the Bygrave equations will yield more accurate results when calculated on a slide rule than the more usual sine/cosine equations.

Given that this is a 10" rule, the full range of sine values = scale ST + scale S = 20 inches. The full range of tangent values = scale ST + scale T_2 + scale T_3 + scale T_4 = 40 inches.

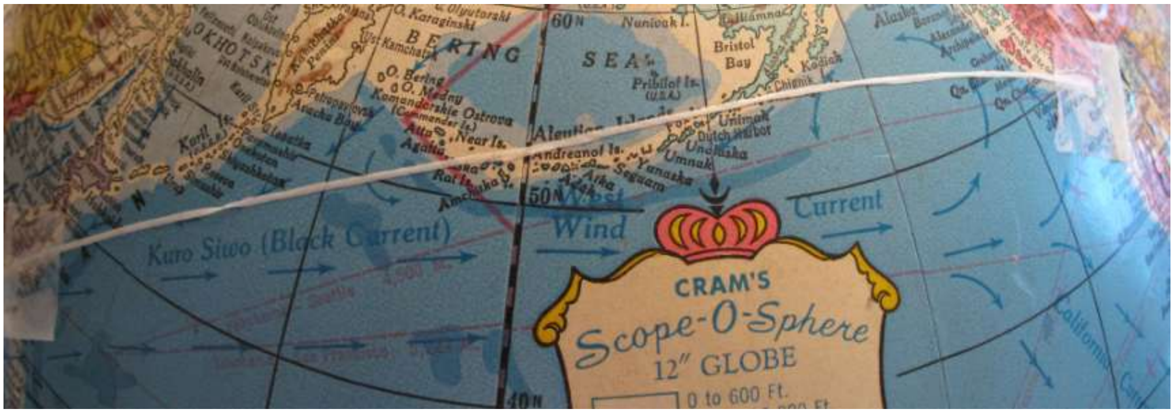
The Bygrave equations still use cosine values, and like sines, all the solutions need to fit into 20" of rule length. But a person can dial in his tangent values more precisely, using all 40" of rule length. So at least *some* of the values you use will be more precisely set than if you are using sine/cosines only.

If you wish to sail from Port Hardy, on the northern tip of Vancouver Island, to Tokyo, your "straight line" course is called a great circle route.



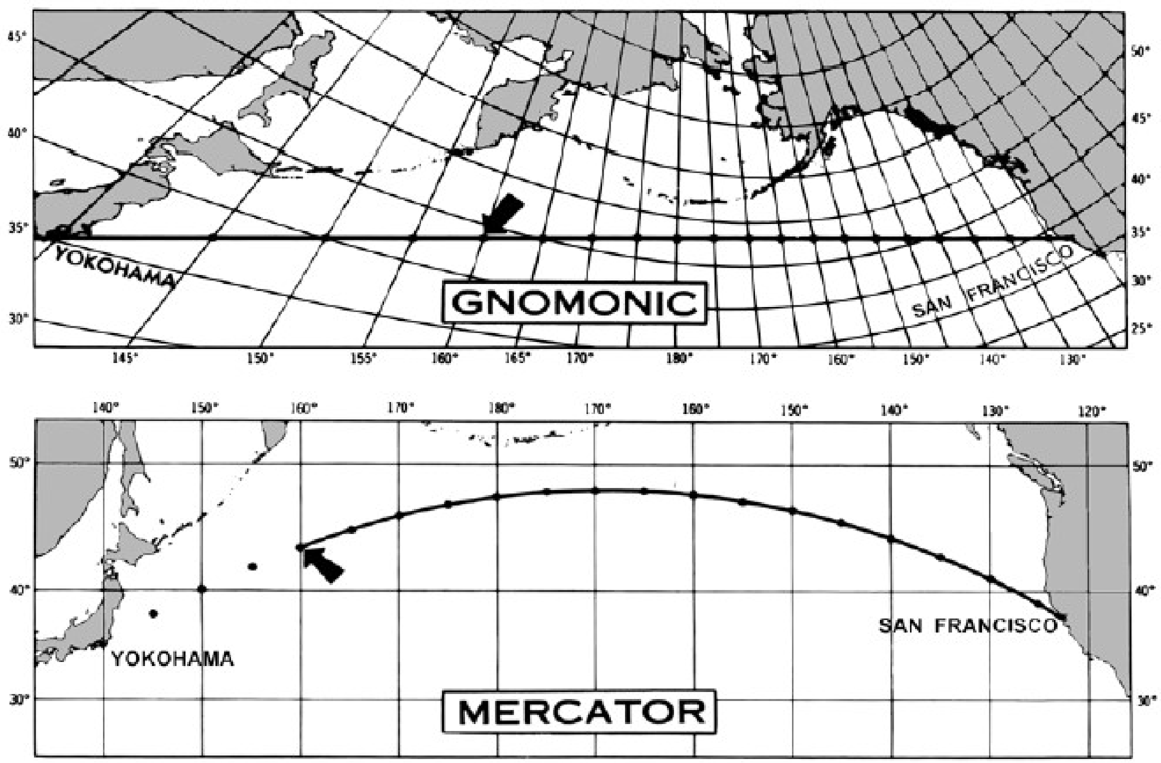
Such a course looks like a curve on a Mercator chart, as above. But if you get a globe and tape a piece of dental floss from your starting port to your destination, you can visualize that the shortest distance from Port Hardy to Tokyo will indeed take you through the Bearing Sea.

I am deeply indebted to Gary for coaching me on how to do great circles/sight reduction with the Bygrave equations.



Figuring out what course to steer to be most efficient can be a bit of a challenge, insofar as the compass course is slowly but continuously changing.

You can purchase a gnomonic chart, of course.

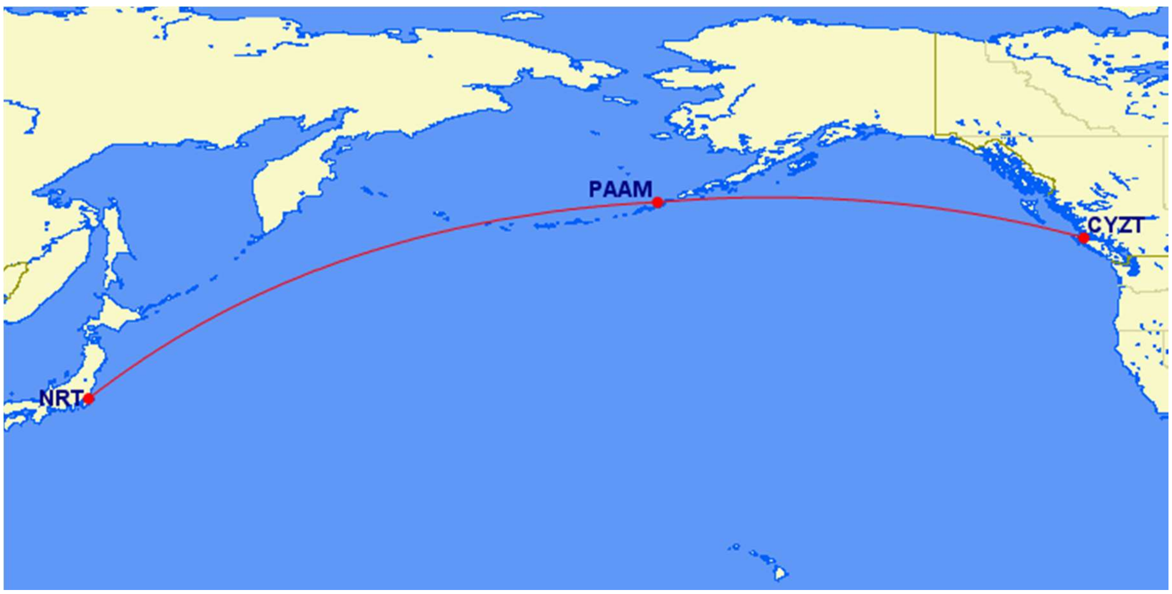


But frankly, I have never much cared for gnomonic charts. They are a cool idea, but costly, and big enough to be awkward to use in a 45 foot yacht. Further, depending on your cruise plan, you may need several of them. Big container ships can pick a course and stick with it across an ocean, but cruising sailors tend to be always working compromises between wind/weather/shortest-course. Depending on the location of the North Pacific High, a sailing yacht on its way from Hawaii to Vancouver may go almost as far north as the Aleutian Islands before picking up the westerly breezes that let it head for home. This can lead to an untidy gnomonic chart as you keep replotting great circle courses from locations you never expected to be at.

On the other hand, I can do a great circle calculation in 4 minutes on a slide rule,[32] once a day, and recalculate an almost[33] perfect great circle route, from anywhere to anywhere.

Besides, as will be seen below, once you figure out how to calculate a great circle route, you will automatically be able to do sight reduction of any star you wish without having to pack all six volumes (at 12.6 pounds!) of Pub. 229 along.[34]

Now, as I am planning my trip from Port Hardy to Tokyo, I discover that Dutch Harbor is almost directly on the great circle route I want to follow, so I decide that I will lay over there for a day or two to warm up, wash some clothes, then get a hamburger and fries after weeks of boat food.



Port of Departure:	Port Hardy, N 50° 43.3', W 127° 29.6'
Destination:	Dutch Harbor, N 53° 53.4', W 166° 32.5'

---

[32] I can also do it with an electronic calculator, of course. But there is no time savings for me. It takes 4 minutes to work out on a calculator, and 4 minutes to work out with a slide rule. It is more *fun* to use the slide rule.

[33] The earth is big enough that a rhumb line course is virtually identical to a great circle course for distances less than 200 nm. So a once-a-day recalculation of the course to sail will work out just fine. Frankly, those who use gnomonic charts often only calculate their course changes for once every 5° of longitude. The man with a slide rule in his hand will steer smaller.

https://www.starpath.com/catalog/books/1898.htm

Because of the ease of use, even a slide rule lover like me will still want to keep at least volumes 2 and 3 of Pub. 249 aboard to use with sun/planets plus a number of the bright stars in the sky.

But for those occasional sights of stars that lie outside the range of declinations that Pub. 249 covers, your Mark 1 Navigator's Slide Rule will serve just fine.

Inputs for calculation: the latitudes of both ports, plus the difference between the longitudes.

```
              166° 32.5′
             -127° 29.6′
Difference     39° 02.9′ West (i.e. from Departure to Dest is west)
```

This difference is known as the “meridian angle” and is abbreviated as “H”.

When subtracting a value of, for example, 23° 58' from 90°, it is easier if you first convert that 90° 00' to 89° 60'…an equivalent value, but easier to work with.

```
   90° 00'                       89° 60'
  -23° 58'                      -23° 58'
Hard to do                       66° 02'
```

Port Hardy (N 50° 43.3′, W 127° 29.6′) to Dutch Harbor (N 53° 53.4′, W 166° 32.5′)

Great Circle Calculations/ Sight Reduction

If using for celestial SR, "Destination" = the GP of the celestial object.
Latitude Destination = Declination of object.
Longitude Destination = GHA.

My Longitude = W (E/W) 127° 29.6′ °

Longitude Destination = W (E/W) 166° 32.5′ °

Meridian Angle = W (E/W) 39° 03′ °

DATA INPUTS: Meridian Angle < 90°	Differences Where Meridian Angle > 90°
Meridian Angle (t) = W (E/W) 39° 03′ °	180°- Meridian Angle = (t) = ___ (E/W) ________ °
My Latitude (L) = N (N/S) 50° 43′ °	
Latitude Destination (d) = N (N/S) 53° 53′ °	

1. tan(d) ÷ cos(t) = tan(W)

   W = 60° 30′ °

2. Use [+W] if d has same sign as L.
   Use [-W] if d has opposite sign as L.

   (90° - L) ± W = X

   X = 99° 47′ °

3. Ignore the sign of X (i.e. -60 = 60).
   If X < 90°, then X = Y
   If X > 90°, then 180 - X = Y

   Y = 80° 13′ °

4. [cos(W) ÷ cos(Y)] * tan(t) = tan(Z)

   Z = 66° 53′ °

5. cos(Z) * tan(Y) = tan(Hc)

   Hc = 66° 17′ °

   Zn = 293° 07′ °

   Convert Hc from decimal deg to deg/min if plotting a celestial position.

   Hc = ____ ° ____ ′

6. (90° - Hc) * 60 = Distance in nm

   Distance = 1,422 nm

2. (... = X

= ________ °

89° 60′
-50° 43′
+60° 30′
99° 47′

Azimuth Rules for Step 5

Meridian angle (t)	1° to 179° W	1° to 179° E
L is in North Latitude		
If d or W > L	Zn = 360 - Z	Zn = Z
if d contrary or W < L	Zn = 180 + Z	Zn = 180 - Z
L is in South Latitude		
If d or W > L	Zn = 180 + Z	Zn = 180 - Z
if d contrary or W < L	Zn = 180 + Z	Zn = 180 - Z

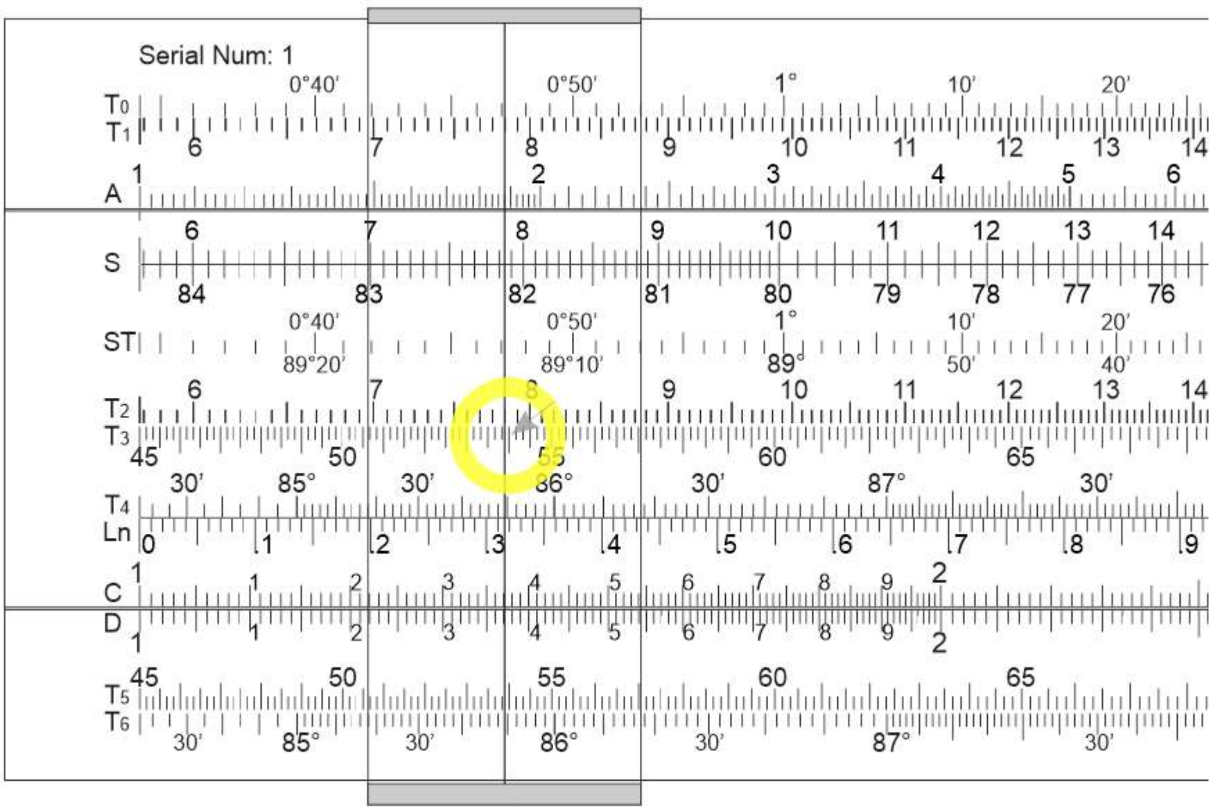
Equation 5 works for sight reduction of all sextant sights, as well as for great circle route calculations. Equation 6 works where the distance to be travelled is no more than 1/4 the circumference of the earth away from starting point.

Write down the data inputs: local hour angle,[35] meridian angle,[36] your latitude, and the latitude of your destination. You will use the left hand side of the form exclusively, since your meridian angle is less than 90°.

Step 1: tan(53° 53') ÷ cos(39° 03) = tan (W)

Center the slide in the body, and position the cursor over 53° 53' on the T_3 scale.

There is no way, with slide rule accuracy, you can do anything about that final "point four" minutes of Dutch Harbor's latitude. Just come as close as you can.



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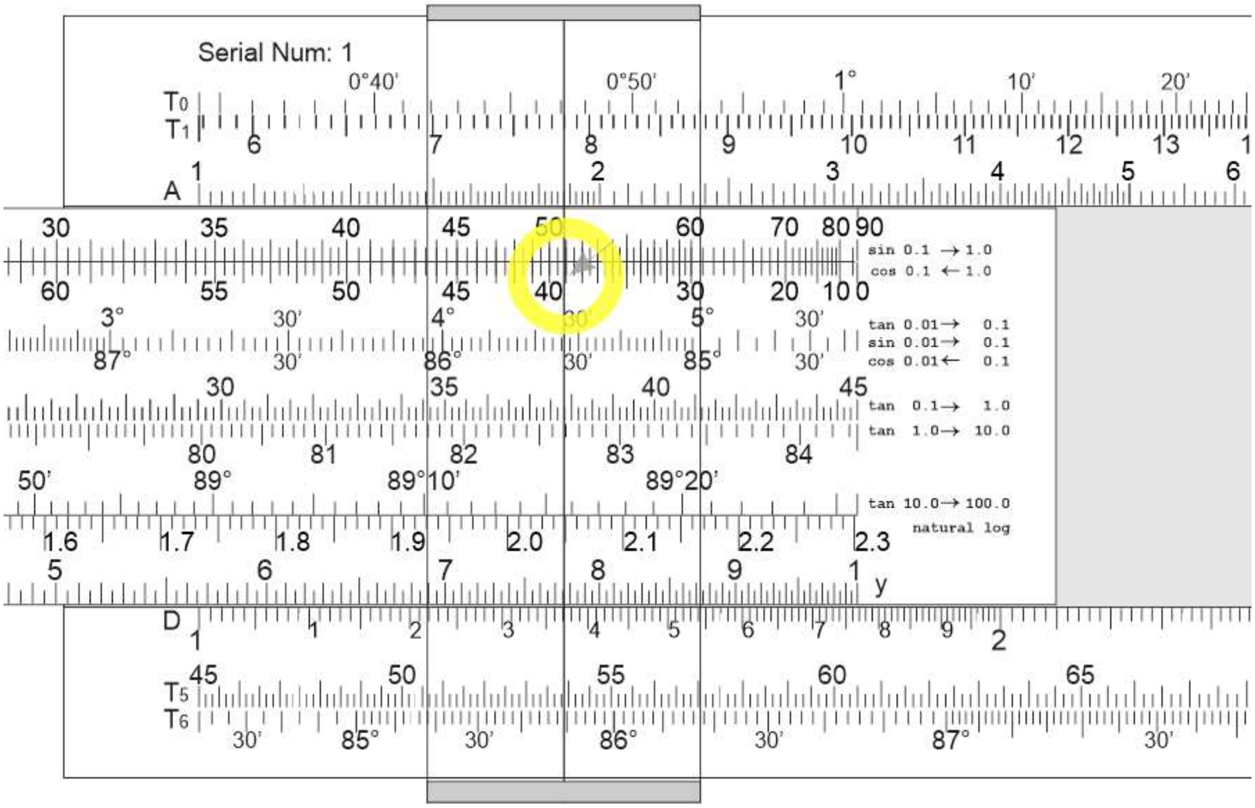
[35] LHA is the difference between your longitude and that of your destination, measured from your location in a westerly direction. So LHA 10° = a meridian angle of 10° W. LHA 350° = a meridian angle of 10° E.

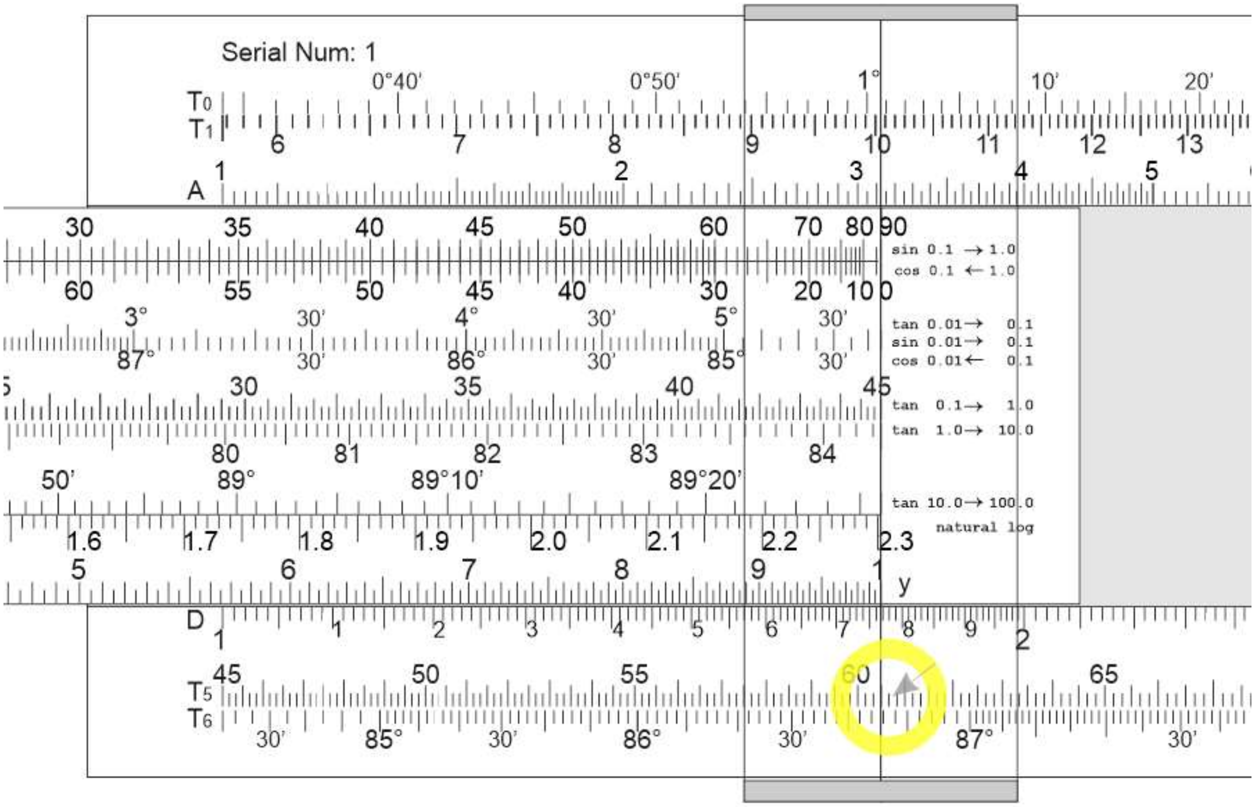
[36] If you are fuzzy on "meridian angle", it is the difference between your longitude and that of your destination, expressed as a value from 0° to 180°, going either east or west. Your location, your destination, and the north pole are the three points of the navigational triangle you are trying to solve.

If your entire training in celestial navigation involved the LHA from 0° to 360°, measured westward, then you have to reorganize your thinking a bit to use either a slide rule or a calculator to solve the triangle.

Position cos(39° 03') from the S scale underneath the cursor, in order to divide.



Move the cursor to the right index, and read 60° 29' on the T_5 scale.

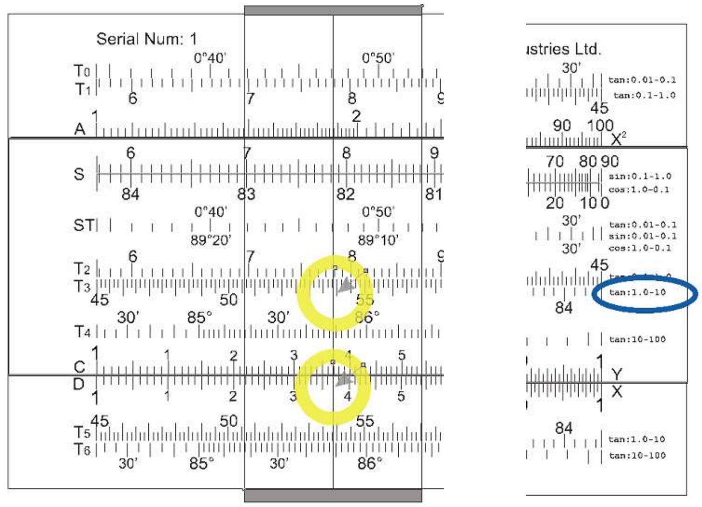


Which of the T-scales should I use to read my answer?

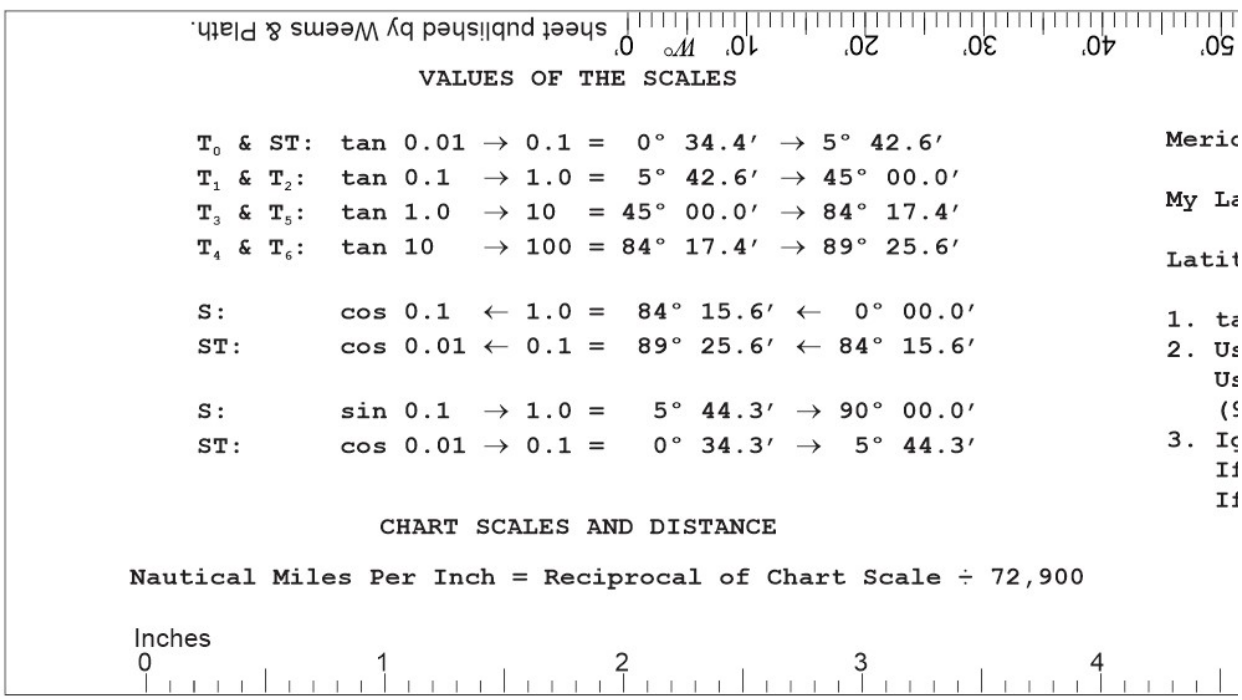
First, if you are doing these calculations once a day while sailing, the values will be in the same general range from one day to the next. Having calculated a great circle route on one day, you already know roughly what to expect when you recalculate the next day.

If you are starting this particular problem cold, however, you can get a sense of the range of your final answer by observing the numeric values of tan(53° 53') and cos(39° 03).

Note the value of tan(53° 53'), and also look at the right end of the T_3 scale.

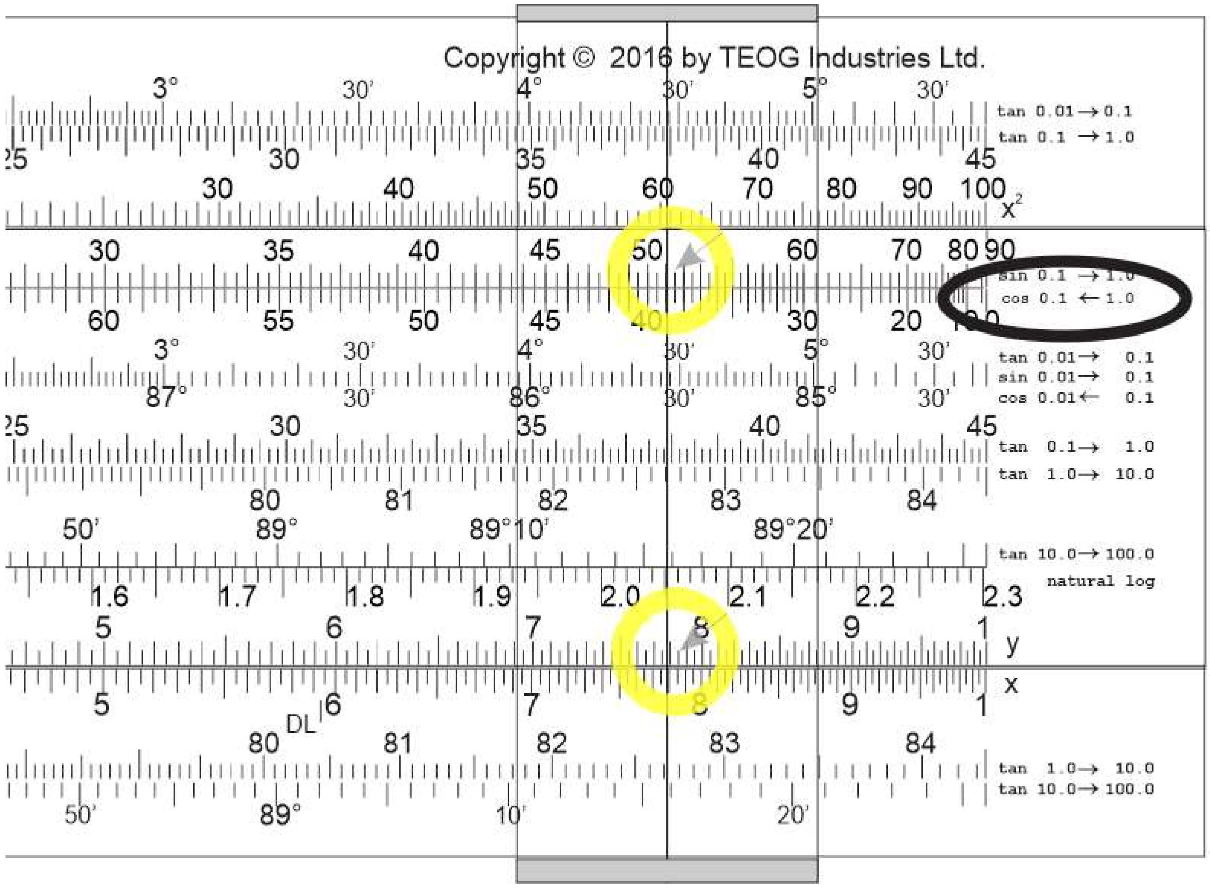


You can also flip the rule over.



You see that scale T3 deals in values of 1.0 to 10. So the tangent of this angle is 1.37.

Look at the value of cos(39° 03'), and observe the cosine values on the right edge of the scale. Scale S deals in cosine values of 1 to 0.1.



So the value of cos(39° 03') is 0.776. This means that the equation of step 1 may be expressed as: 1.37 ÷ 0.776. I can know at a glance that .776 is going to go into 1.37 something in the neighborhood of 2 times.

Since scale T_5 deals in values from 1 to 10, I know that my answer of 2-ish needs to be read on scale T_5.

Once you get a feel for the kinds of answers that are in-range, you can multiply and divide cosines and tangents without concern for the values on the C or D scales.

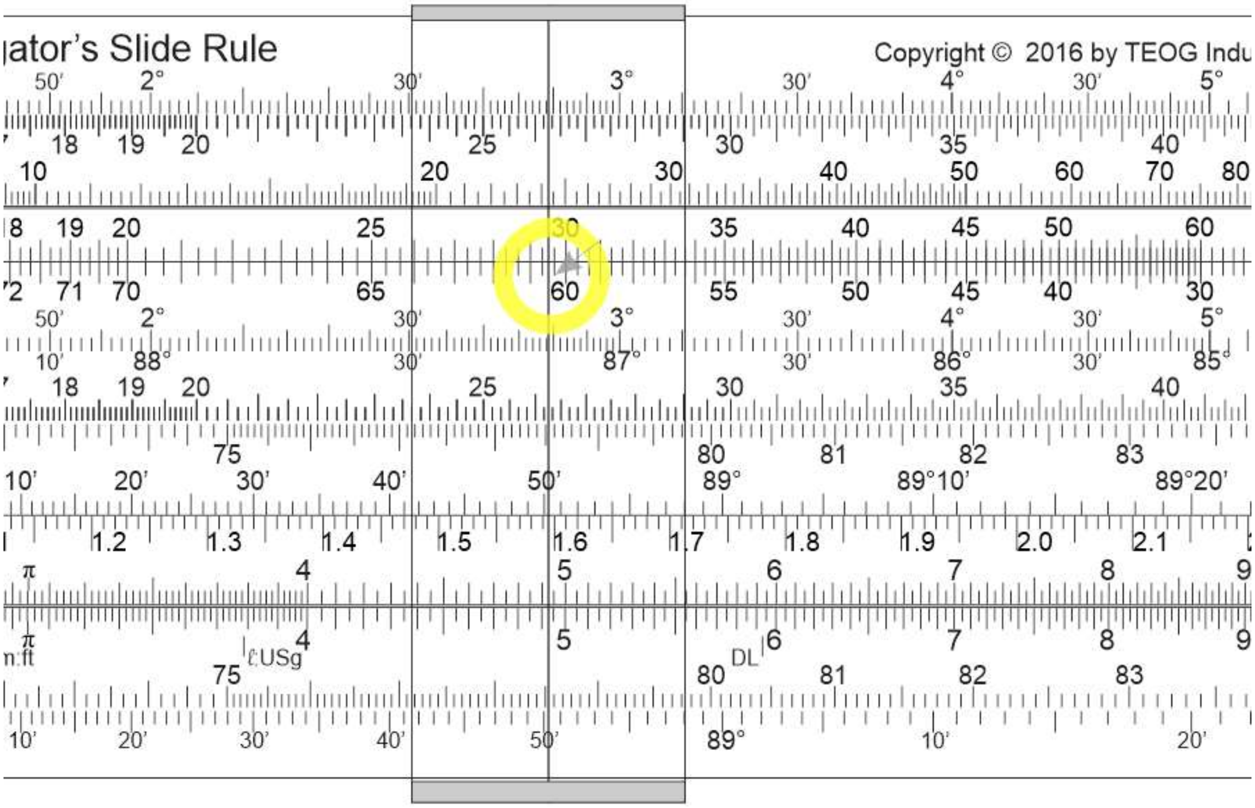
Section *§50* reviews the technique for doing mental math with trig functions.

Step 2: (89° 60' - 50° 43') + 60° 29' = 99° 46'

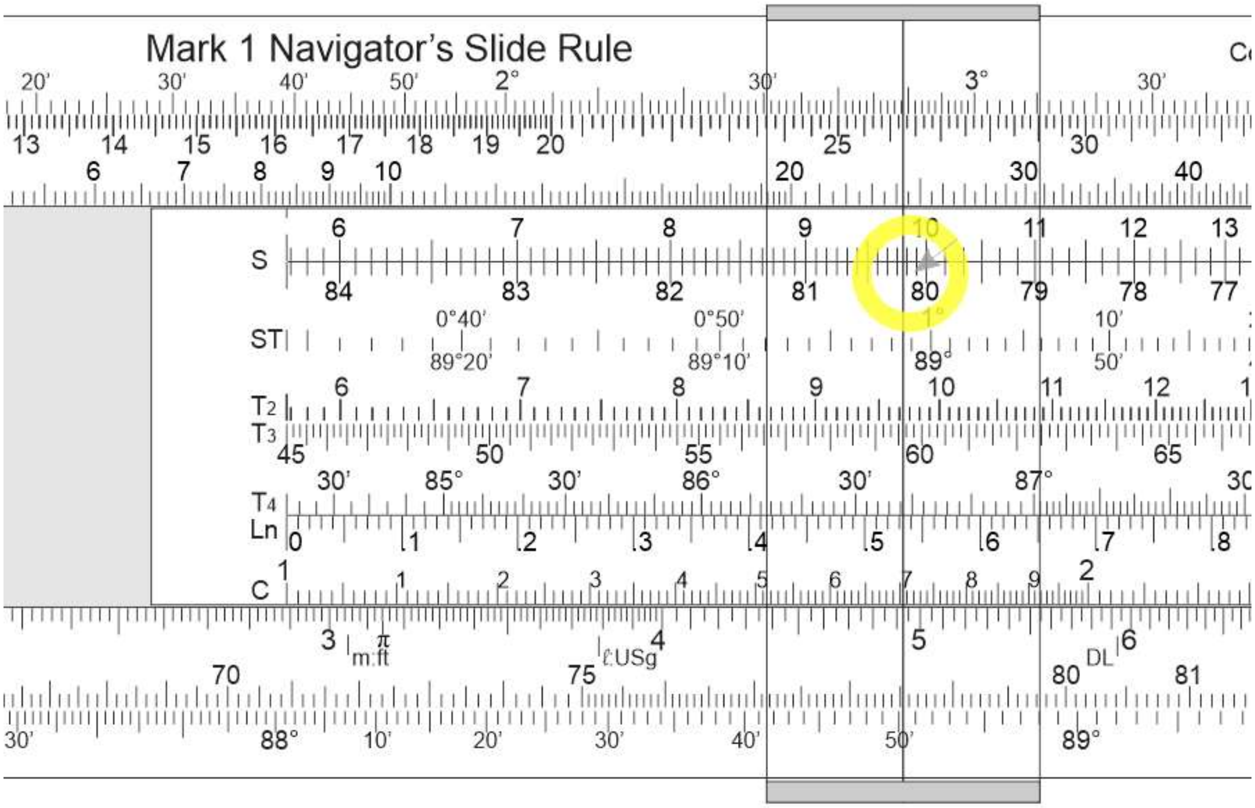
Step 3: 179° 60' – 99° 47' = 80° 13'

Step 4: [cos(60° 30') ÷ cos(80° 13')] * tan(39° 03') = tan(Z)

Center the slide, then position the cursor over cos(60° 30') on the S scale.

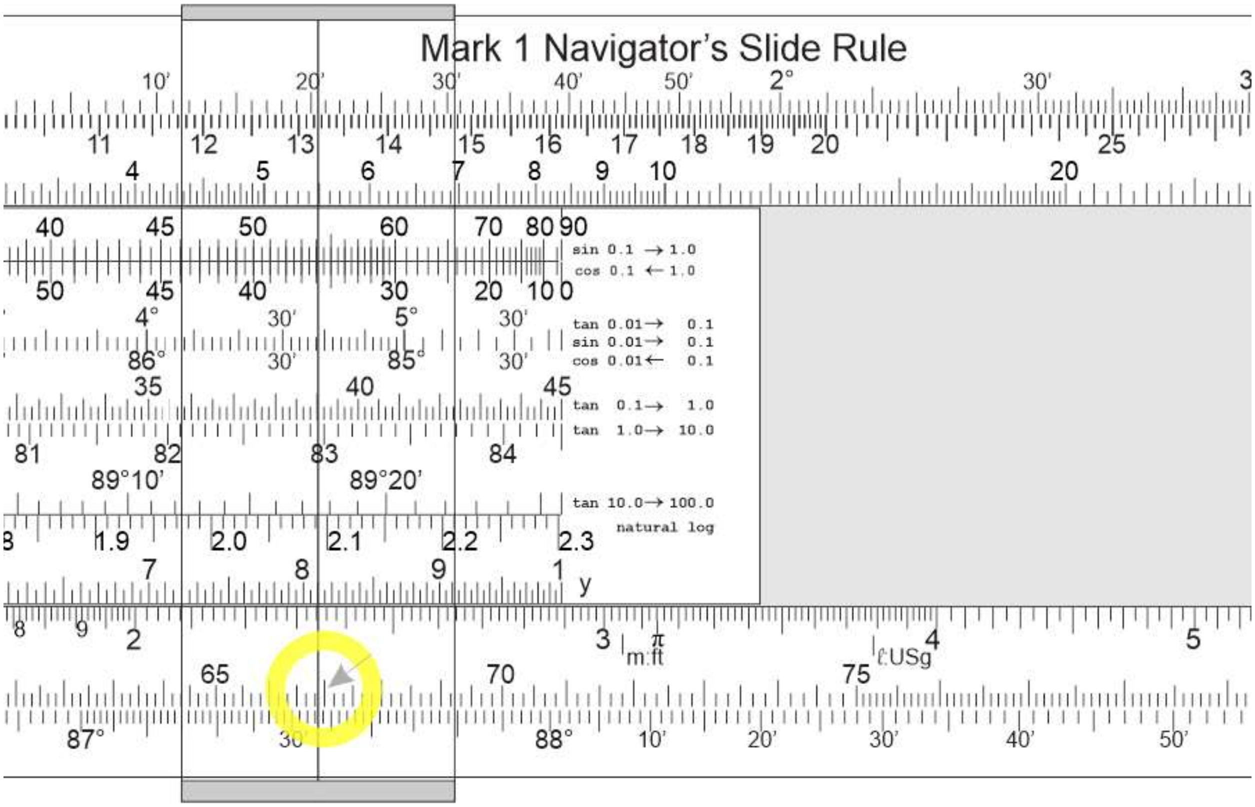


Move cos(80° 13') on the S scale beneath the cursor. This performs your division.



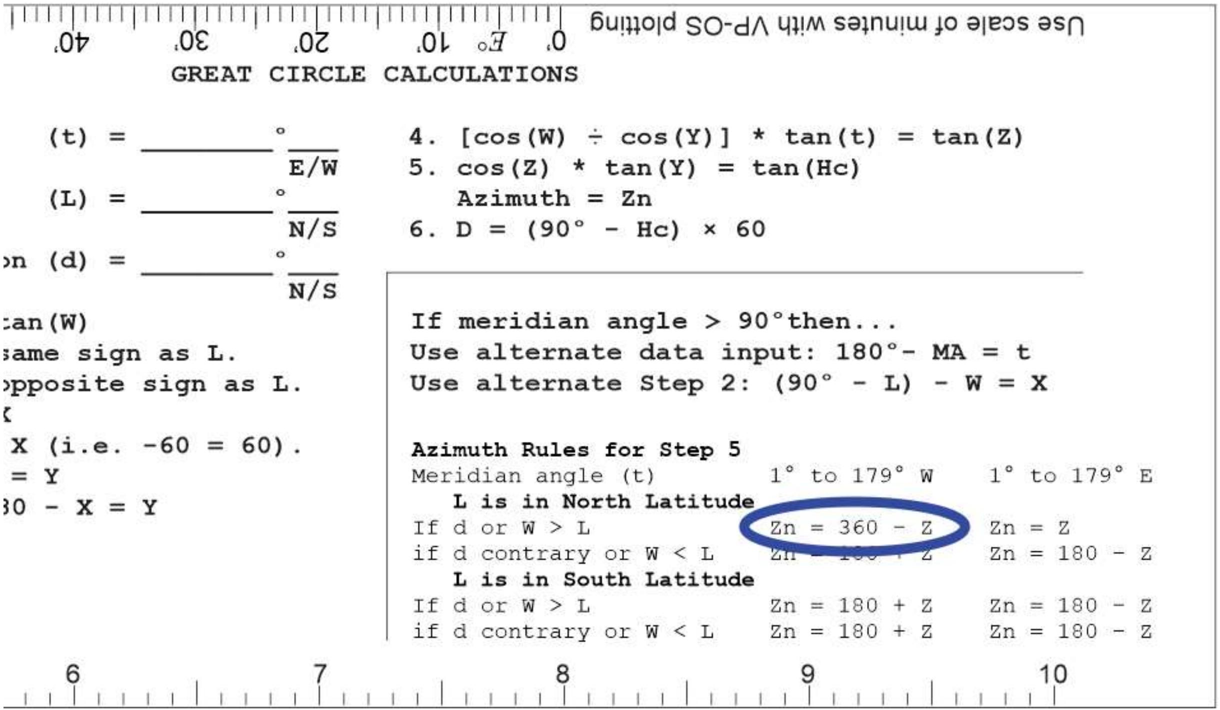
You are ready to go straight to multiplying now, so move the cursor over the top of tan(39° 03') on the T_2 scale...except you can't quite do that now. Tan(39) extends out too far to the right. So move the cursor to the left index, then move the right index on the slide to the cursor, and THEN multiply by tan(39° 03') from the T_3 scale, and read your answer on the T_5 scale:

Z = 66° 53'

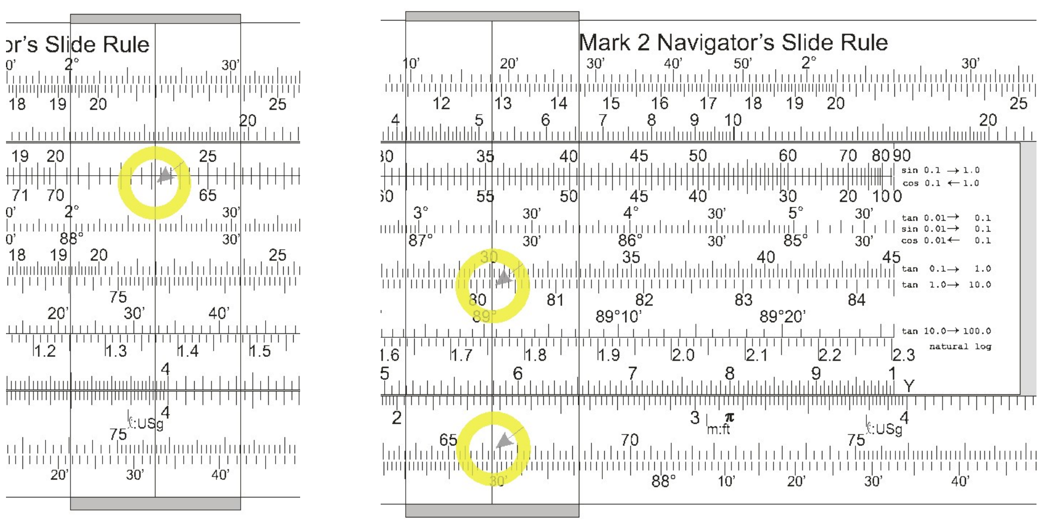


Convert Z to Zn using the chart on the worksheet or on the back of the rule. In this case, L is north of the equator, and d is greater than L (for that matter, W is also greater than L), and the LHA is 39°. So Zn = 360° - 66° 53'.

Remember that your math will work out more easily if you think of this as
359° 60' – 66° 53'.



Step 5: Center your slide, and position the cursor over cos(66° 53') on the S scale. Then move the right index on the slide under the cursor, and multiply by moving your cursor over tan (80° 13')

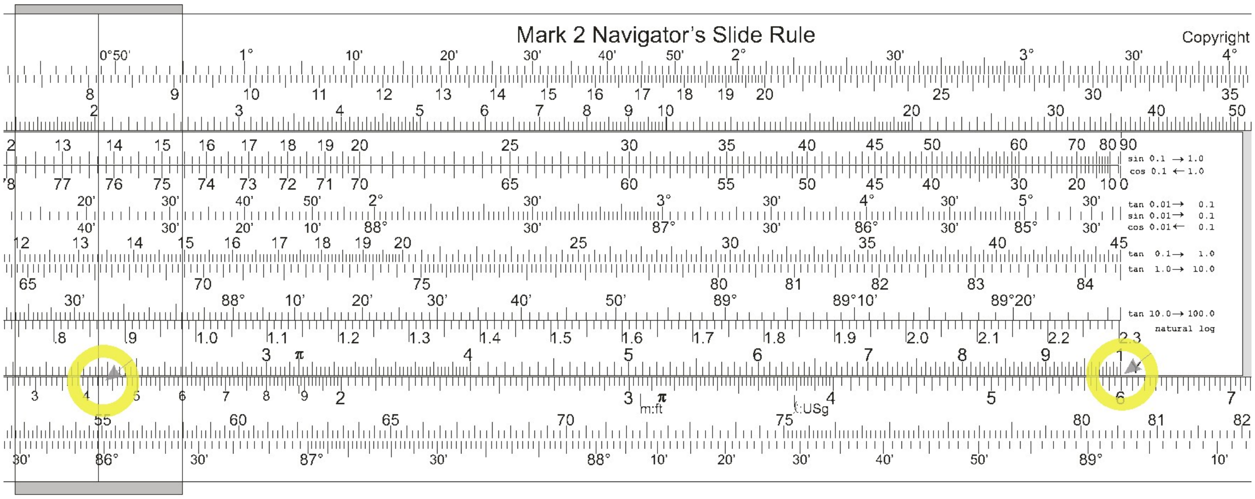


Hc = 66° 17'

Zn = 292° 42'

Step 6: 89° 60' – 66° 17' = 23° 43'. Convert this to decimal degrees = 23.7°

Multiply by 60, and get 1,422 nm.



§284.1 How accurate are the results in problem #1? Are the differences navigationally significant?

Paul Hirose has done a statistical analysis of the accuracy of a 10" slide rule using these equations (the "Bygrave equations") at http://fer3.com/arc/m2.aspx/Bygrave-formula-accuracy-10-inch-slide-rule-Hirose-jul-2009-g8985

He programmed a computer to simulate common slide rule errors, and found that on average, a slide rule solution could be depended on – on average – to deliver an Hc altitude to within 1.7' of an electronic calculator, and to deliver an azimuth within 2.0'.

Zn, azimuth, the course to steer your boat, differs by 0° 3.6' between slide rule and computer calculation for this particular problem. Given that small boats have a tough time steering any smaller than ±5°, obsessing over azimuth accuracy of 3.6' is taking months off your life for no gain at all.

So no, the azimuth difference is not navigationally significant in this specific solution.

Hc differs in this problem by 3.6 nm between slide rule and computer calculation (this is the "Intercept"). This is not a very large error when the total distance is 1,422 nm.

So the distance difference is also not navigationally significant in this specific solution.

One more example will give you a chance to practice a bit more with your slide rule, as well as to illustrate the sight reduction of a star that has a meridian angle of more than 90°.

Schedar is a star in the circumpolar constellation Cassiopeia, and so is visible from mid-north latitudes all night. When you are looking south, no celestial object has a meridian angle of more than 90°. That is where your horizon is. But in looking at circumpolar stars, you can see a group of stars that clusters about the North Pole through a full 360°. So then, the meridian angle of circumpolar stars could be anything up to east or west 180°.

This star has a declination (which you will use as input value "d" in the work form) of N 56° 37.0'. The sidereal hour angle ("SHA" – equivalent to the longitude of a destination when doing a great circle course calculation) could be anything, depending on the time and date when you sight it.

For purposes of this example, we will assume that your vessel is still at Port Hardy, and that the SHA is such that the meridian angle from your location is 120° east (LHA = 240°).

Sight of the star Schedar from Port Hardy

Great Circle Calculations/ Sight Reduction

If using for celestial SR, "Destination" = the GP of the celestial object.
Latitude Destination = Declination of object.
Longitude Destination = GHA.

My Longitude = ___ (E/W) ________°

Longitude Destination = ___ (E/W) ________°

Meridian Angle = E (E/W) 120° 00′°

DATA INPUTS: Meridian Angle < 90°	Differences Where Meridian Angle > 90°
~~Meridian Angle (t) = ___ (E/W) ________~~°	180°- Meridian Angle = (t) = E (E/W) 60° 00′°
My Latitude (L) = N (N/S) 50° 43′°	
Latitude Destination (d) = N (N/S) 56° 37′°	

1. tan(d) ÷ cos(t) = tan(W)

W = 71° 46′°

~~2. Use [+W] if d has same sign as L.~~
~~Use [-W] if d has opposite sign as L.~~

~~(90° - L) ± W = X~~

~~X = ________~~°

2. (90° - L) - W = X

X = -32° 29′°

3. Ignore the sign of X (i.e. -60 = 60).
If X < 90°, then X = Y
If X > 90°, then 180 - X = Y

Y = 32° 29′°

4. [cos(W) ÷ cos(Y)] * tan(t) = tan(Z)

Z = 32° 43′°

5. cos(Z) * tan(Y) = tan(Hc)

Hc = 28° 09′°

Zn = 32° 43′°

Convert Hc from decimal deg to deg/min if plotting a celestial position.

Hc = ____° ____'

6. (90° - Hc) * 60 = Distance in nm

Distance = ________ nm

Azimuth Rules for Step 5

Meridian angle (t)	1° to 179° W	1° to 179° E
L is in North Latitude		
If d or W > L	Zn = 360 - Z	Zn = Z
if d contrary or W < L	Zn = 180 + Z	~~Zn = 180 - Z~~
L is in South Latitude		
If d or W > L	Zn = 180 + Z	Zn = 180 - Z
if d contrary or W < L	Zn = 180 + Z	Zn = 180 - Z

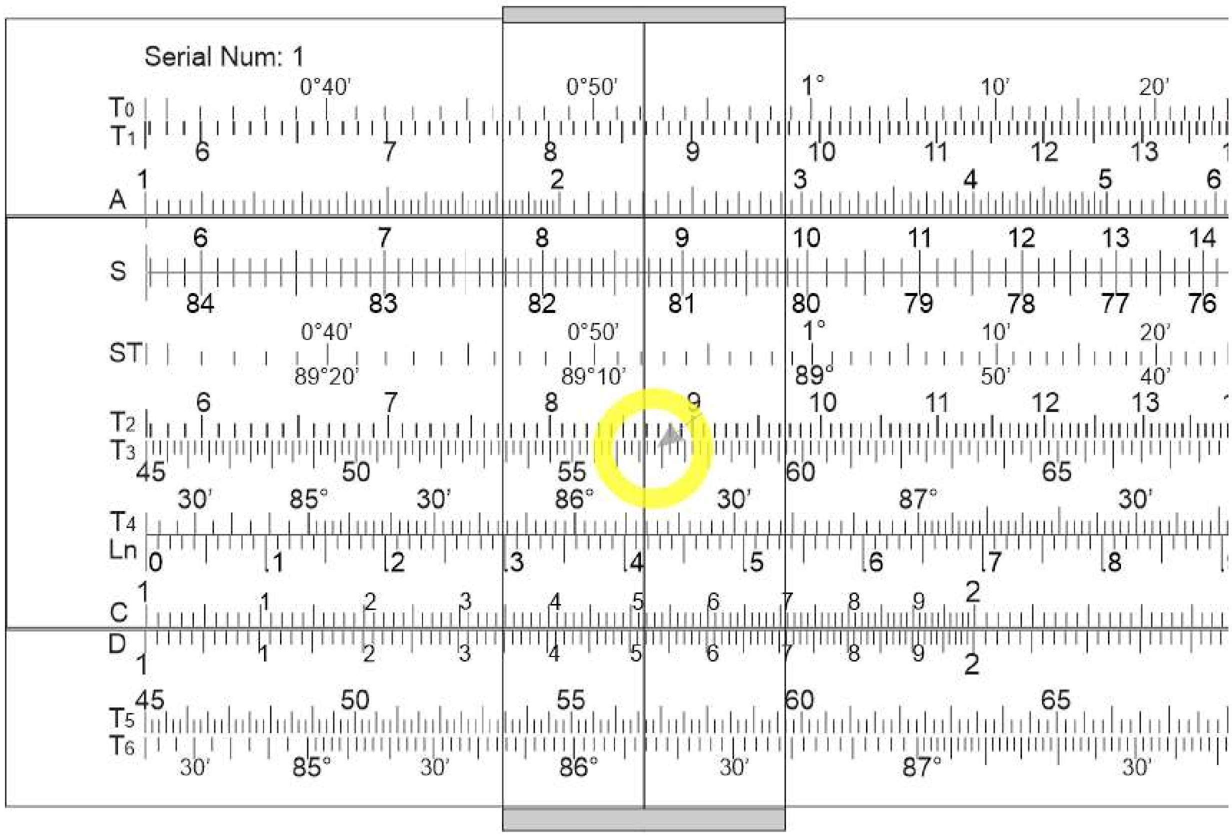
Equation 5 works for sight reduction of all sextant sights, as well as for great circle route calculations. Equation 6 works where the distance to be travelled is no more than 1/4 the circumference of the earth away from starting point.

Since you are dealing with a meridian angle greater than 90°, you will use the substitutions from the right side of the form.

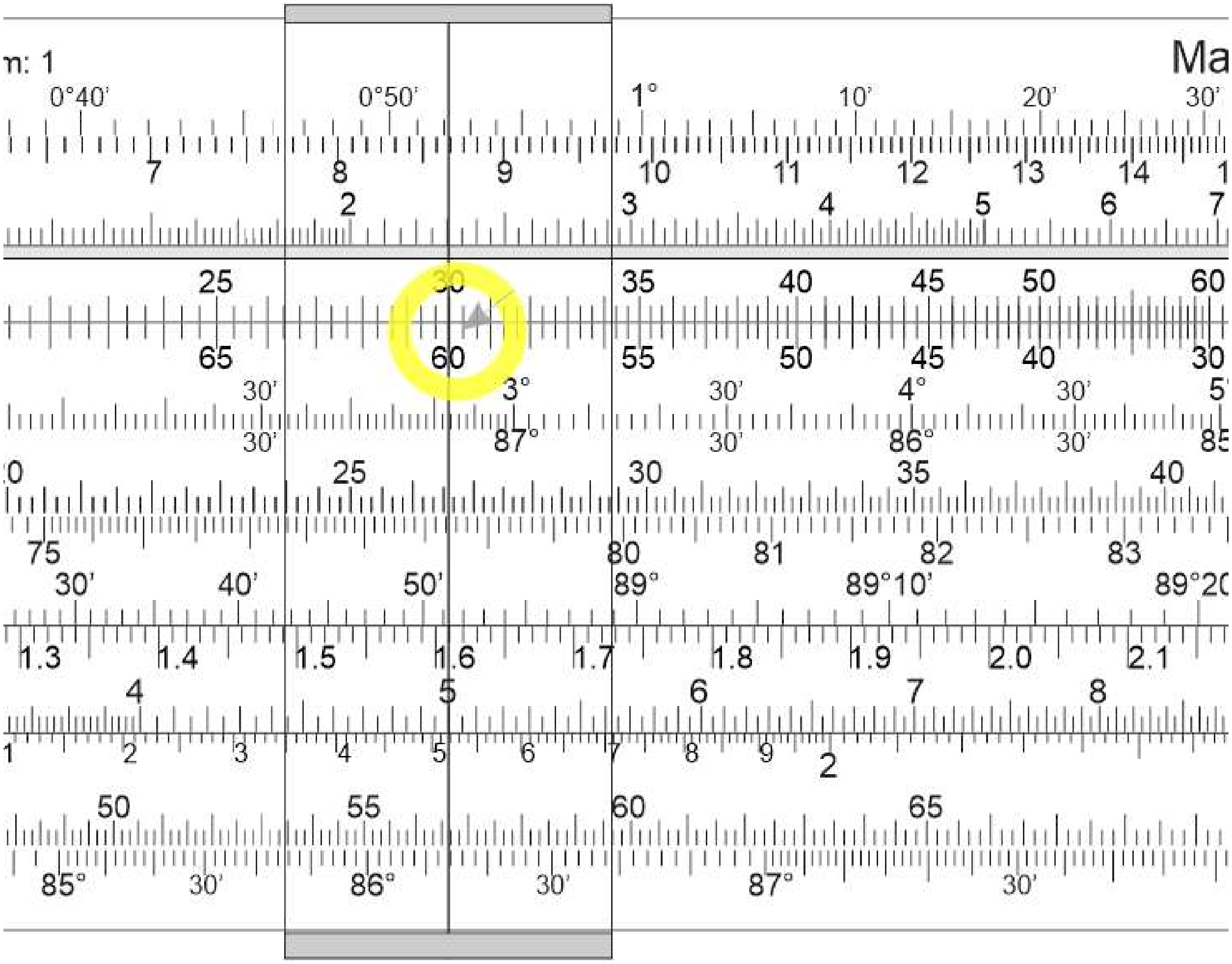
With a meridian angle of 120° E, H becomes 60° E.

Step 1: tan(56° 37') ÷ cos(60°) = tan(W)

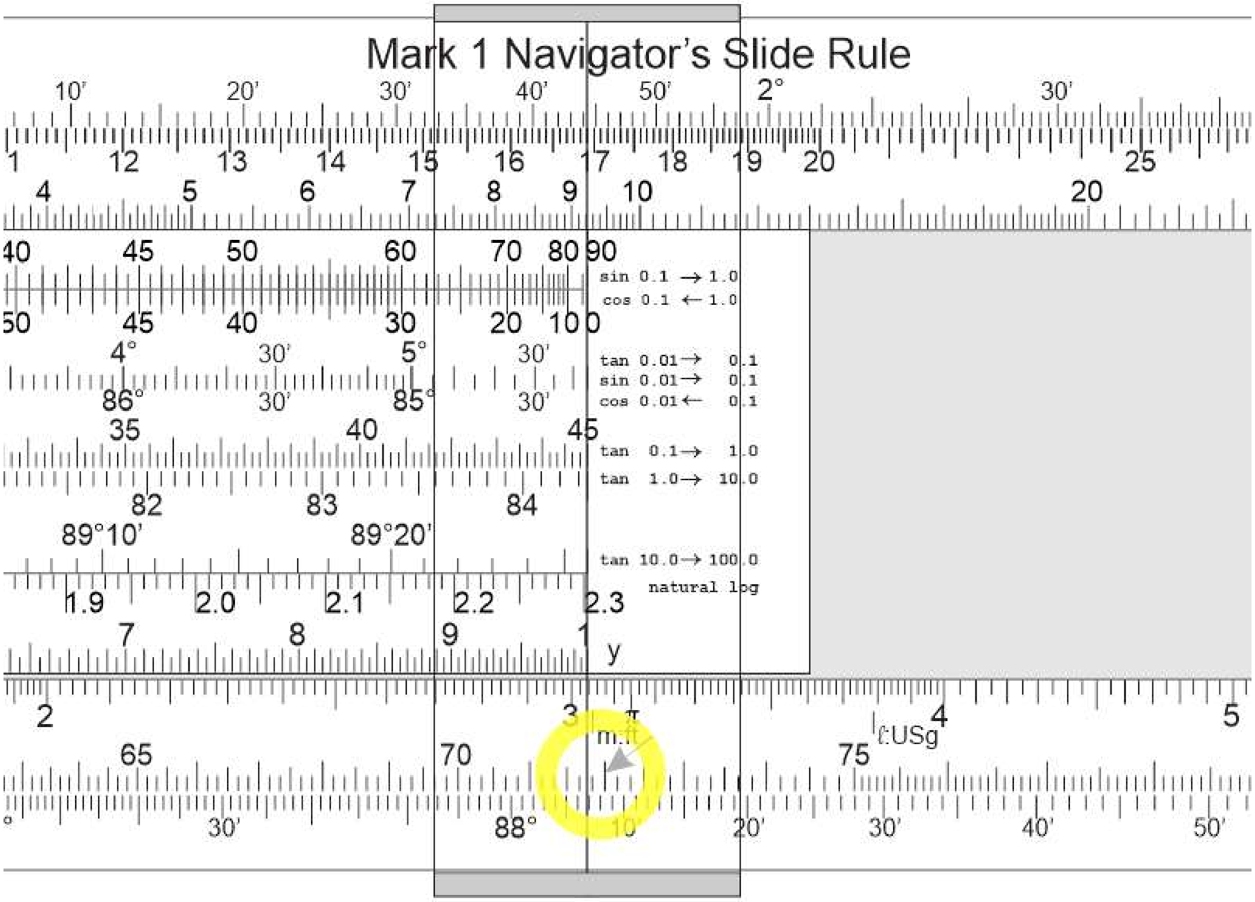
Center the slide. Position the cursor over tan (56° 37') on the T_3 scale.



Position cos(60°) on the S scale directly beneath the cursor.



Move the cursor over the right index. Read the answer off the T_5 scale.



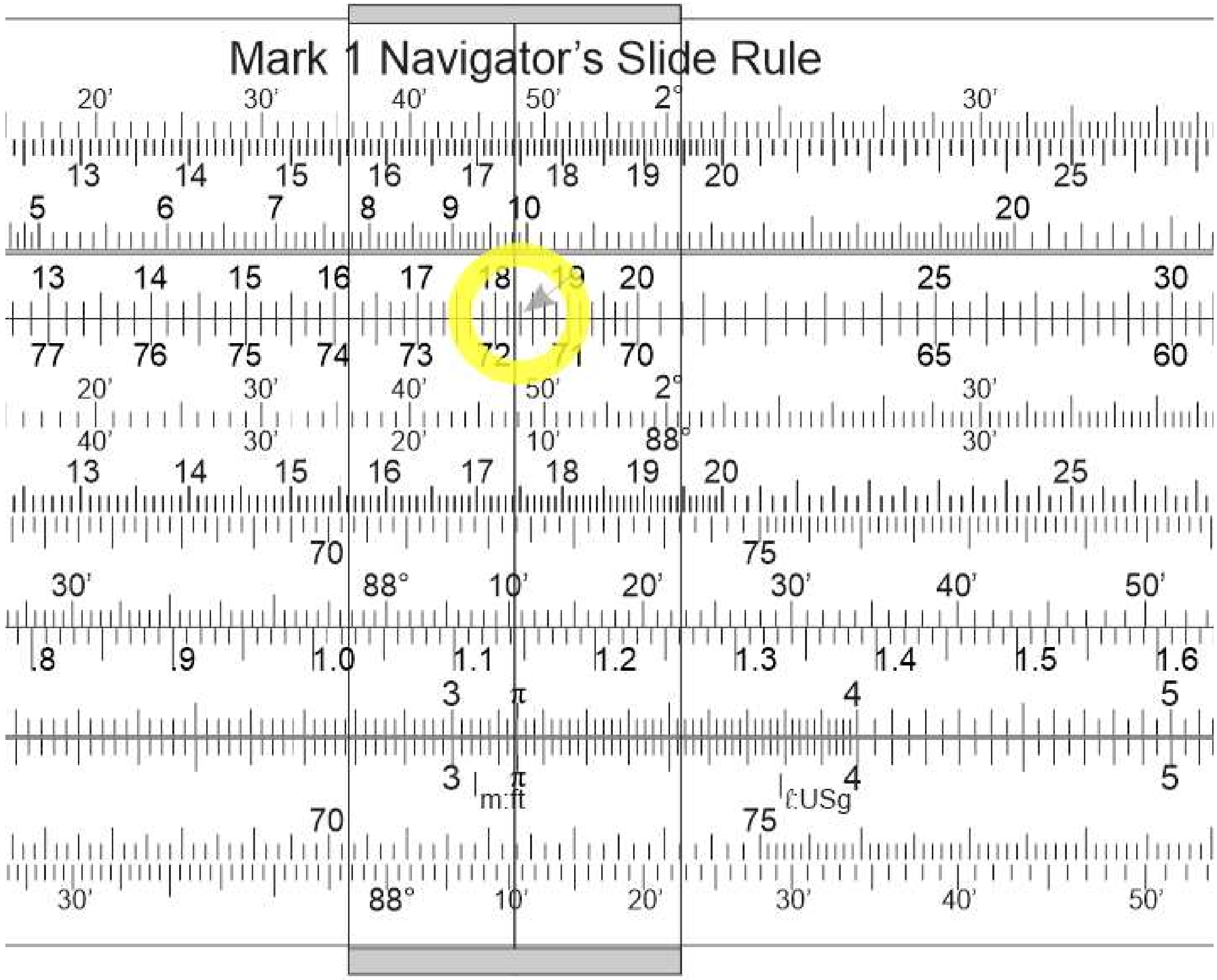
Step 2: Since you have been dealing in a meridian angle greater than 90°, complete step 2 from the right hand column. I find that doing minus minus values is a bit confusing.

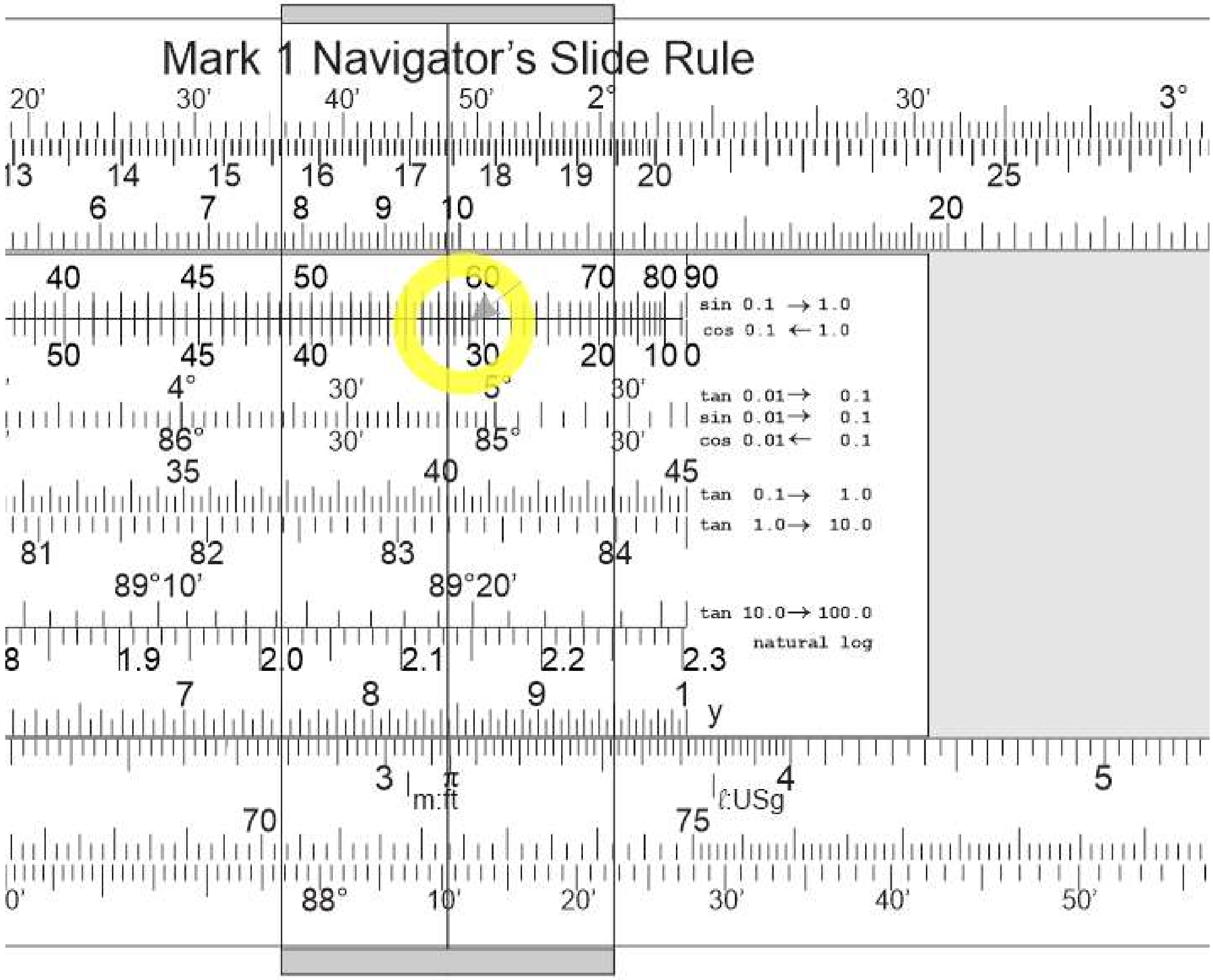
What I did was to add L and W together = 122° 29', and then subtract 90° 00' from that. This is particularly suitable in that I am to ignore the sign of "X" in step 3.

Step 3 is simple. Just enter 32° 29'.

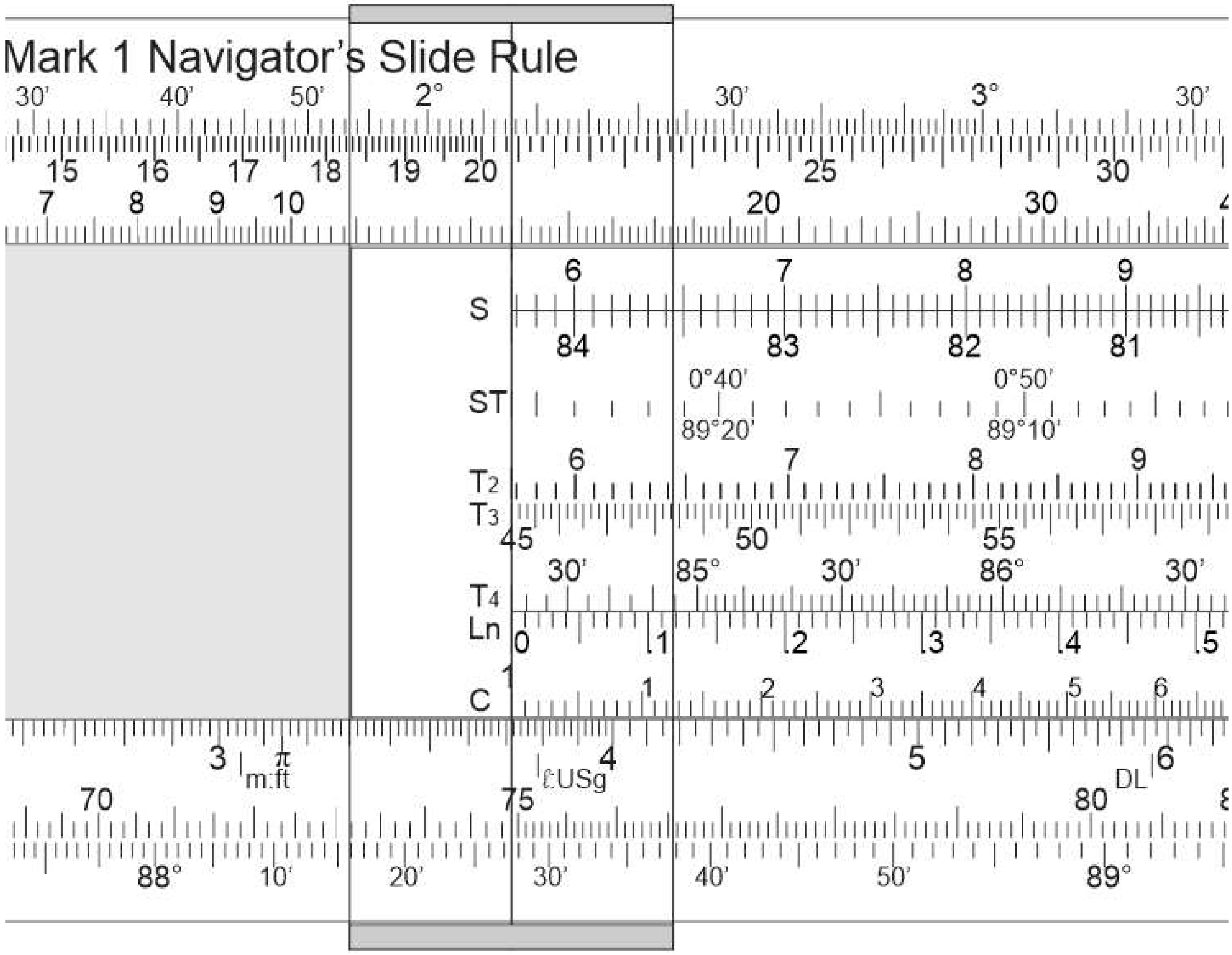
Step 4: [cos(71° 46') ÷ cos(32° 29')] * tan(60° 00') = tan(Z)

Position your cursor over cos(71° 46') on the bottom half of the S scale, then move the slide until cos(32° 29') on the S scale is beneath the cursor. This will set up the division portion of step 4.

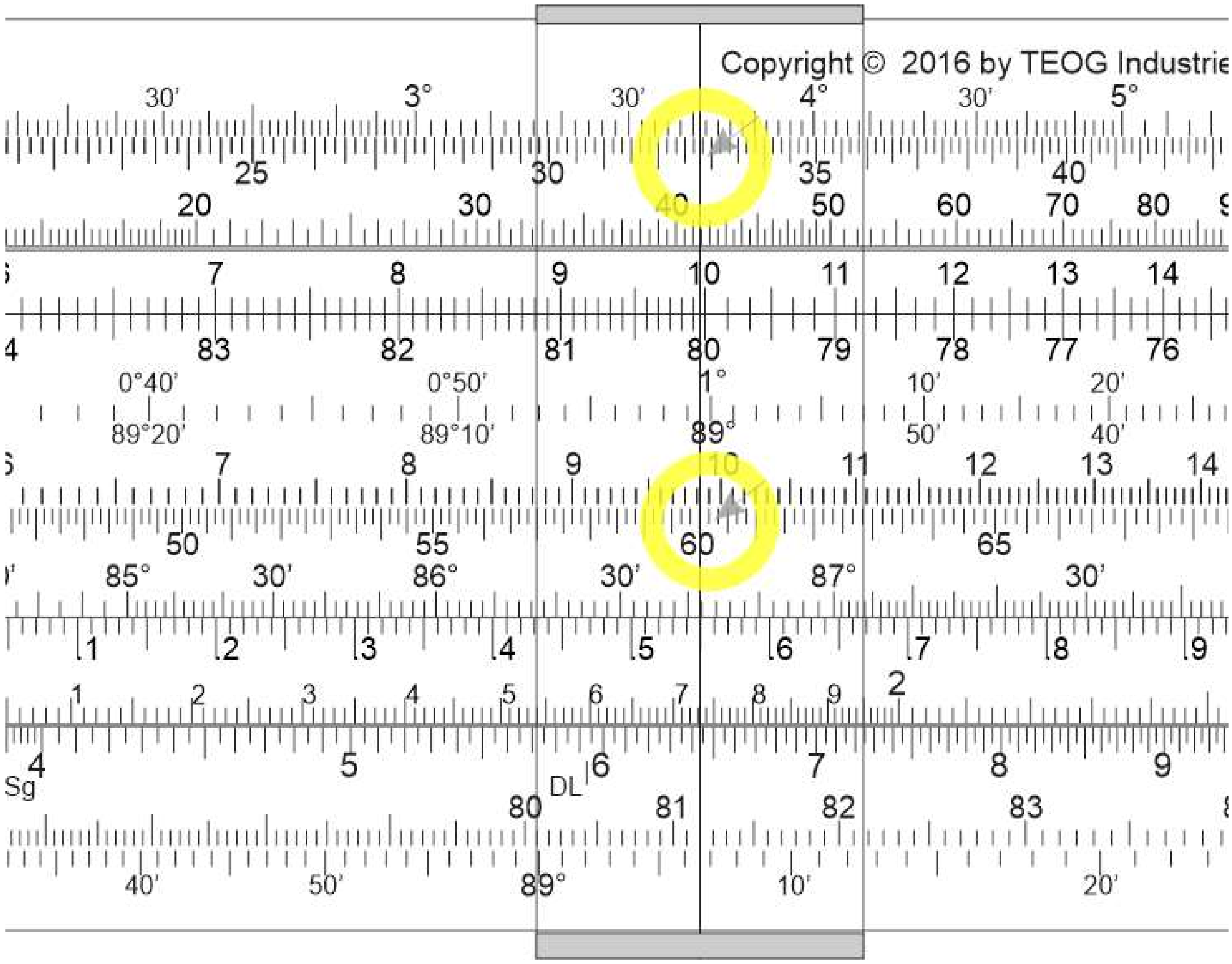

Mark 1 Navigator's Slide Rule
m:ft
ℓ:USg



You cannot simply slide the cursor until it is over tan(60°). You must first move the cursor over the right index, then move the slide until the left index is under the cursor.

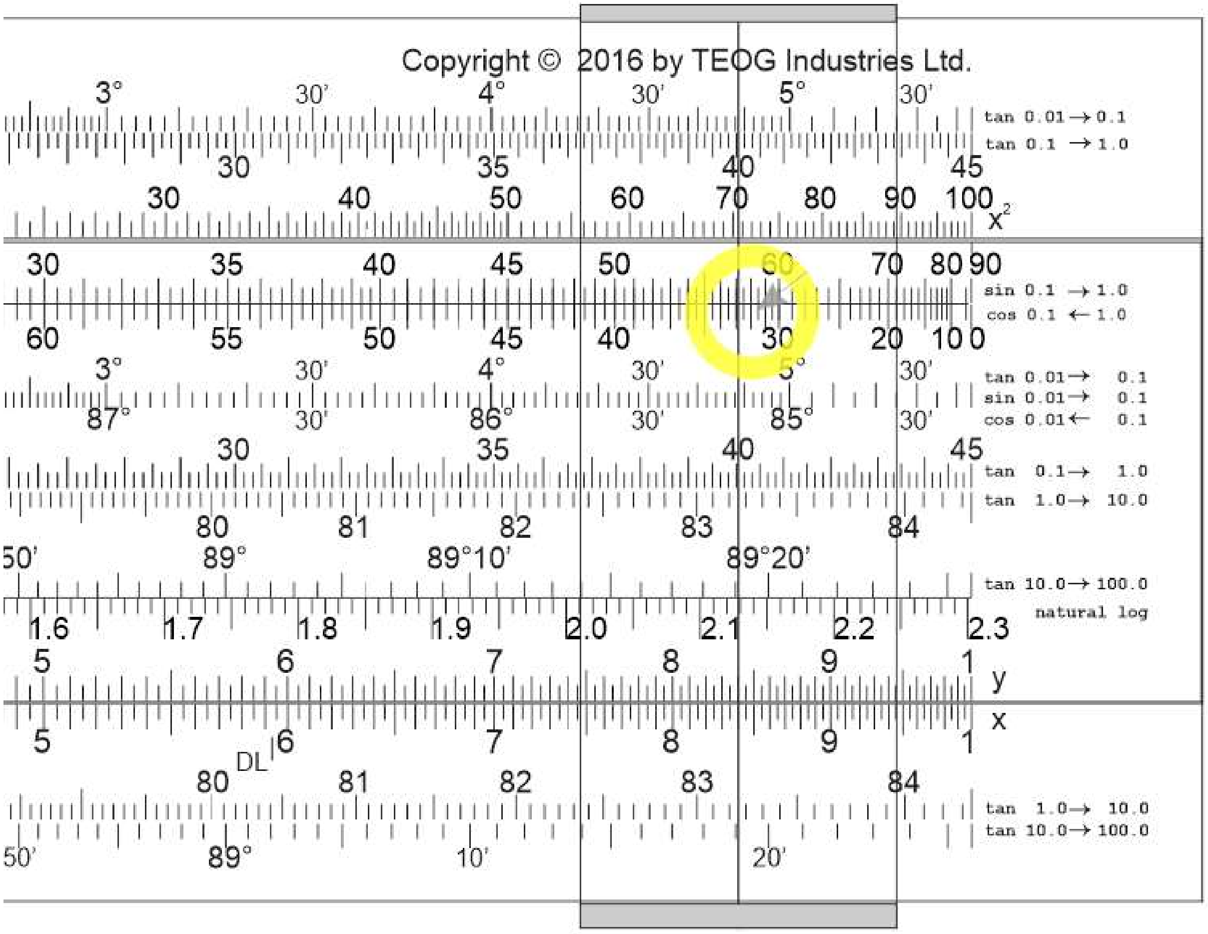


Now slide the cursor over the top of tan(60°) on the T_3 scale. Read the answer of 32° 43' on the T_5 scale.

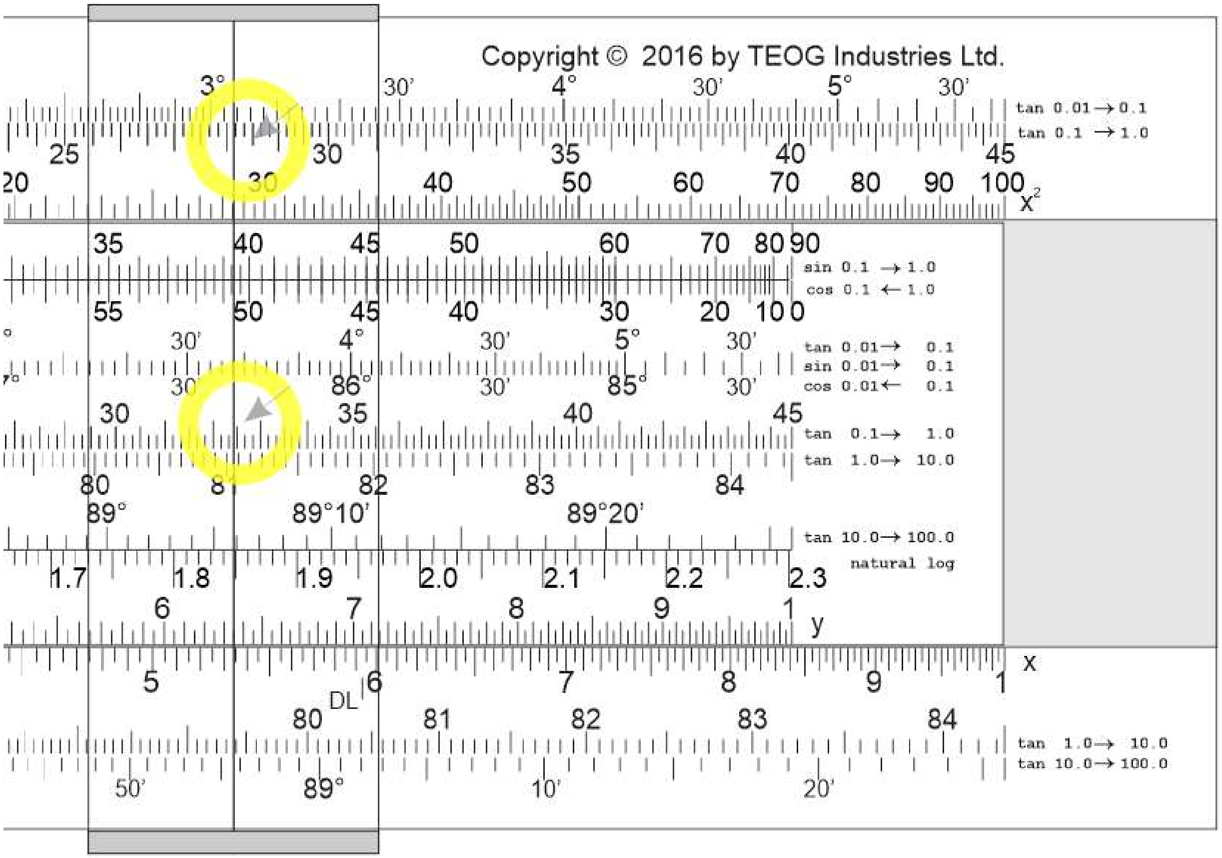


Step 5: cos(32° 43') * tan(32° 29') = tan(Hc)

Center the slide. Move the cursor over the top of cos(32° 43') on the S scale.



Move the right index under the cursor. Move the cursor until it is over tan(32° 29') on the T_2 scale. Read the answer of 28° 09' off the T_1 scale.


Copyright © 2016 by TEOG Industries Ltd.
tan 0.01→0.1
tan 0.1 →1.0
x²
sin 0.1 →1.0
cos 0.1 ←1.0
tan 0.01→ 0.1
sin 0.01→ 0.1
cos 0.01← 0.1
tan 0.1→ 1.0
tan 1.0→ 10.0
tan 10.0→100.0
natural log
y
x
DL
tan 1.0→ 10.0
tan 10.0→100.0

§284.2 How accurate are the results in problem #2? Are the differences navigationally significant?

These equations, when worked on an electronic calculator, give you "gold standard" results. I use RealCalc for Android, a calculator app that will allow you to enter angles either as decimal degrees or as degrees and minutes.

If you are working in degrees and minutes, you can enter decimal minutes (e.g. 28° 10.3'), but RealCalc will instantly convert that into degrees, minutes, and decimal seconds (were 1 arc second = 1/60th of a minute).

If you wish to toggle this type of display to decimal degrees, you can do that by touching the <° ' "> key.

As I do these equations, the calculator works them out to 10 decimal places. I store each of the results in a separate memory location, so that I can refer back to them (that is, most of the values are used in at least two equations; storing them makes sure I do not have to rework an equation to use a value a second time). Here you can see where I stored t, L, d, W, Y and Z.

According to this, the slide rule's intercept for this problem is only 1.3 nm from the electronically-calculated value, and the azimuth agrees to the nearest minute.

What can we say about this?

> The standard I use for navigational significance is "Can I find Bermuda using my celestial navigation techniques?"
>
> The highest point in Bermuda is 260 feet above sea level. From a small yacht, you will see Bermuda on the horizon as long as you are no further than 20 nm away. In a pre-GPS era, more than one sailor simply missed Bermuda, and sailed on to Europe without ever seeing this small island group.
>
> If you have a sextant error of 4 nm, and a slide rule, sight-reduction error of 1.3 nm, you will have no trouble finding Bermuda. So then, I would say that the slide rule error of 1.3 nm in this problem is not navigationally significant.
>
> If you are sailing in the Pacific and navigating between volcanic islands, then celestial navigation is even more forgiving. The highest point in Tahiti is 7,352 feet above sea level. You absolutely cannot miss it as long as you have even the remotest idea of what you are doing with your sextant and slide rule.

As soon as you make a landfall, you will put your sextant back in its box and begin to use piloting techniques, including hand-bearing compass, depth sounder, and radar, to guide you into port.[37]

A slide rule will give more accurate results on some problems than on others. There are certain ranges of values where a slide rule can hardly be used, while other ranges exist where it is inherently more accurate.

You can see on your slide rule that tangent values are compressed in the neighborhood of 45°, while cosine values are dramatically compressed at less than 10°. You need to be aware of these ranges as you evaluate the likely accuracy of your slide rule work.

Paul Hirose, in the statistical analysis mentioned above,[38] found that 95% of slide rule solutions have an Hc that is accurate to within 3.7'.

Using a slide rule is a mechanical skill, and you will probably get slightly better results as you practice more.

If your slide rule sticks at all, you may gain more accuracy by using a non-petroleum lubricant, such as powdered graphite – available for sale from most automotive supply shops.

The two examples above illustrate:

- Northern-hemisphere-to-northern-hemisphere, meridian angle *less than* 90°
- Northern-hemisphere-to-northern-hemisphere, meridian angle *greater than* 90°.

These equations work without modification where you are looking at:

- Northern-to-southern-hemisphere (or vice versa), or
- Southern-to-southern-hemisphere.

The only thing you need to keep track of are the rules for turning Z into Zn.

```
- - - - - - - - - - - - - - - - - - - - - - - - - - -
Azimuth Rules for Step 5
LHA*                        1° to 179°       181° to 359°
     L is in North Latitude
If d or W > L             Zn= 360 - Az      Zn= Az
If d contrary or W < L    Zn= 180 + Az      Zn= 180 - Az
     L is in South Latitude
If d or W > L             Zn= 180 + Az      Zn= 180 - Az
If d contrary or W < L    Zn= 360 - Az      Zn= Az
- - - - - - - - - - - - - - - - - - - - - - - - - - -
```

[37] It is possible to use your sextant in piloting, but to learn about that you do not need a user manual on slide rule use.
[38] http://fer3.com/arc/m2.aspx/Bygrave-formula-accuracy-10-inch-slide-rule-Hirose-jul-2009-g8985

§444.3 Limitations of Navigational Trigonometry – Special Rules

The accuracy of slide rule calculations breaks down if the scenario you are solving for involves any angles that:

- are less than 1° or greater than 179°
- are between than 89° and 91°
- yield azimuths greater than 85°.

At least as pertains to great circle/sight reduction calculations, there are special rules you can follow to cope with these limitations.[39] But in these cases, I think it is time to fire up RealCalc, or even a GPS app, on your Android Phone, love of traditional navigation notwithstanding.

---

[39] Look at the link to the Bygrave instructional form at Gary LaPook's website at http://tinyurl.com/BygraveInstructions for "Special Instructions".

You will find his entire discussion of the Bygrave method, which was designed originally to be used with a cylindrical slide rule, quite interesting at: http://tinyurl.com/GaryLaPook. See also https://en.wikipedia.org/wiki/Bygrave_slide_rule

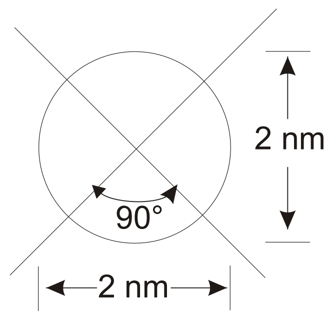
§45. Error Ellipse in Celestial Fix [40]

It is up to the navigator to decide on the size of his error circle if two LOPs cross at 90°, based on his assessment of the most likely accuracy of his sextant work under current conditions.

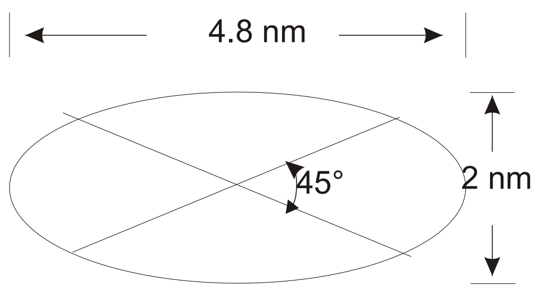
If the two LOPs cross at some angle θ (i.e. difference in azimuths), less than 90°, the circle becomes an ellipse. The ratio of the width of the ellipse (estimated by the navigator) and the length of it, which gets longer as the size of θ decreases, is:

Long Axis of Ellipse = Short Axis of Ellipse ÷ tan(θ/2)

So then, if you take a sight where you think your error is likely ± 1 nm, if your LOPs cross at 90° you would draw your error ellipse as a perfect circle, 2 nm in diameter, centered on the point where your LOPs cross.



If the LOPs cross at 45° then the ratio becomes tan(45°/2) = 0.414.
2 nm ÷ 0.414 = 4.8 nm.



If the LOPs cross at 20°, then the ratio becomes tan(20°/2) = 0.176.
2 nm ÷ 0.176 = 11.3 nm.

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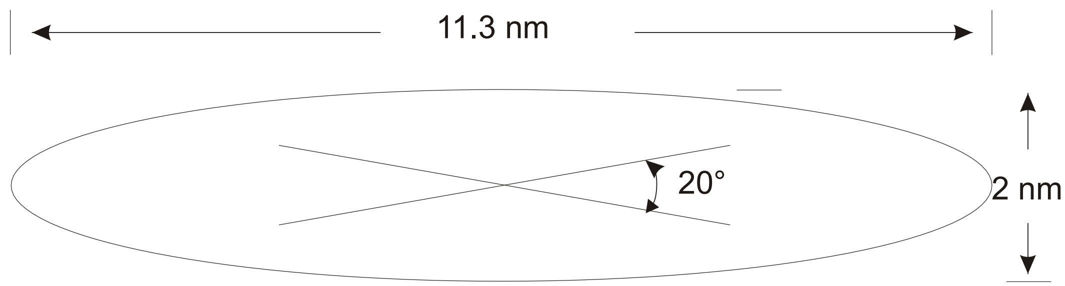
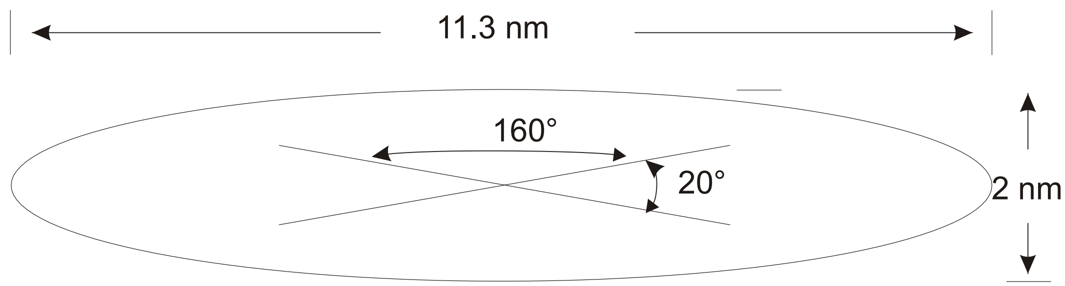
[40] This is from Frank Reed, http://fer3.com/arc/m2.aspx/Error-ellipse-proportions-FrankReed-jun-2017-g39195, retrieved June 10, 2017.



θ should always be less than 90°. Treat a θ of 160° as equivalent to θ = 20°.

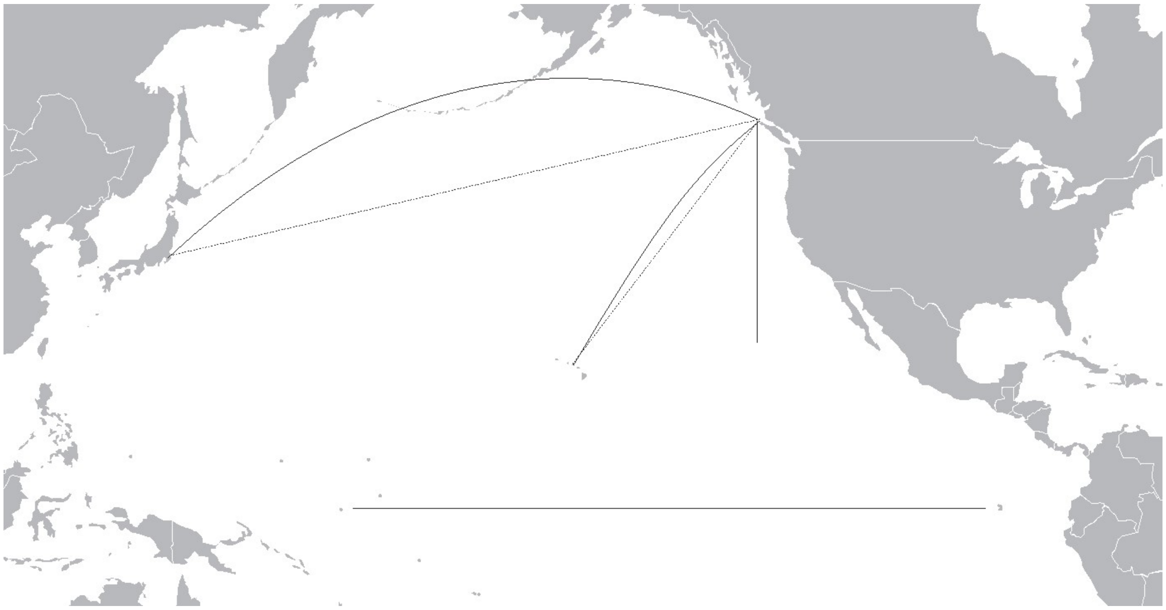


As for fixes that use more than two LOPs and generate a cocked hat, Greg Rudzinski has said, "Any additional LOPs less than the largest azimuth difference will not change the ellipse size but will only serve to shift the ellipse center position and rotate the length axis slightly."[41]

§46. Rhumb Line Route Calculation

When you are travelling N-S from anywhere, or E-W at the equator, a rhumb line course is precisely the same as a great circle route. Great circle routes become progressively more efficient than rhumb line courses the further away you are from the equator, and the more you are travelling east-west rather than north-south. This graphic shows rhumb line courses as dashed lines and great circle routes as solid lines.

[41] Greg Rudzinski, http://fer3.com/arc/m2.aspx/Error-ellipse-proportions-Rudzinski-jun-2017-g39212, retrieved June 12, 2017.



If you are travelling from Port Hardy, BC, to Hawaii, the rhumb line course is little more than an hour of sailing time longer than the great circle route. Definitely not worth the effort of recalculating your great circle heading on a day by day basis.

But if you are seeking to sail non-stop from Port Hardy to Tokyo, the great circle route can shave *several days* off your trip compared to the rhumb line course.

So you know, intuitively, that some cruise plans will benefit from great circle planning, while others will not. Back up your intuitions with solid data by calculating a route first by the great circle method, then by rhumb line. Compare the distance to be travelled. This will help you determine which approach you want to use for a given cruise.

The example below deals in the same Port-Hardy-to-Dutch-Harbor problem as in the first great circle example of section §44.

Port Hardy (N 50° 43.3', W 127° 29.6') to Dutch Harbor (N 53° 53.4', W 166° 32.5')

Rhumb Line Course Calculation

1	Destination latitude	53° 53.4' N		
2	Destination latitude in decimal degrees		53.9° N	
3	My latitude	50° 43.3' W		
4	My latitude in decimal degrees		50.7° N	
5	Difference between destination latitude and my latitude in decimal degrees (i.e. ΔL)		3.17°	
6	Meridian angle between my longitude and destination longitude (i.e. t)	39° 02.9'		
7	Convert t into decimal degrees.		39.17°	
8	π * t in decimal degrees			122.9
9	(destination latitude in decimal degrees ÷ 2) + 45	71.92°		
10	Convert value from line 9 into degrees and minutes	71° 55'		

11	Ln of tan(result of line 10)		1.12	
12	(my latitude in decimal degrees ÷ 2) + 45	70.35°		
13	Convert value from line 12 into degrees and minutes	70° 21'		
14	Ln of tan(result of line 13)		1.03	
15	Subtract line 14 from line 11		0.09	
16	(result of line 15) * 180			16.2
17	Divide value from line 8 by value in line 16			7.58
18	atan(value of line 17) = Z of course as __ ___° ___' __			N82° 29'W
18	Rhumb line course Zn			277° 31'
19	60 * (ΔL from line 5) ÷ cos(Z from line 17) = rhumb line distance in nm			1,450 nm

If your destination is generally northeast of you, your azimuth is going to be "North some-number-of-degrees East". If it is generally southeast of you, your azimuth is going to be "South some-number-of-degrees East". And so on.

While the sight reduction worksheet above uses a chart to help convert from azimuth angles (Z) into azimuths (Zn), when working with rhumb lines you need to keep in mind that azimuth angles are typically prefixed with a cardinal direction, and have a suffix indicating the direction from the cardinal angle to count degrees.[42]

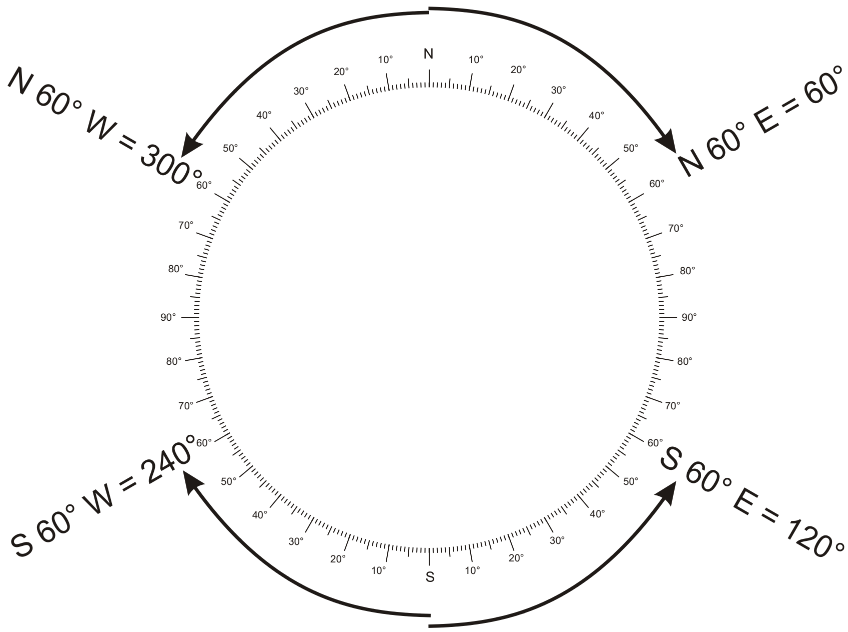
Azimuth Angle (Z)	Azimuth (Zn)
N 10° E	10°
S 10° E	170°
S 10° W	190°
N 10° W	350°

When doing spherical trigonometry, results for the course to steer are in azimuth angles. One must convert these angles into azimuths in order to get a compass course to steer.

In writing down values, write down values to the nearest minute. Ignore the tenths of a minute. There is no way a slide rule can resolve a number down to the nearest tenth of a minute.

[42] William Bligh was a failure as a leader of men, but a superb navigator. You can read his fascinating account of the 3,600 nm voyage in an open boat that he undertook, with a handful of men, after being set adrift by the mutineers of the *Bounty*, available at https://www.gutenberg.org/cache/epub/15411/pg15411.txt.
One of the interesting things to note is that he never uses azimuths to describe his course. Rather, he will talk in terms of sailing "a N 72° W course" (i.e. 288°) or "course S 88° W" (i.e. 268°).

§28.1 Converting an Azimuth Angle into a True Bearing



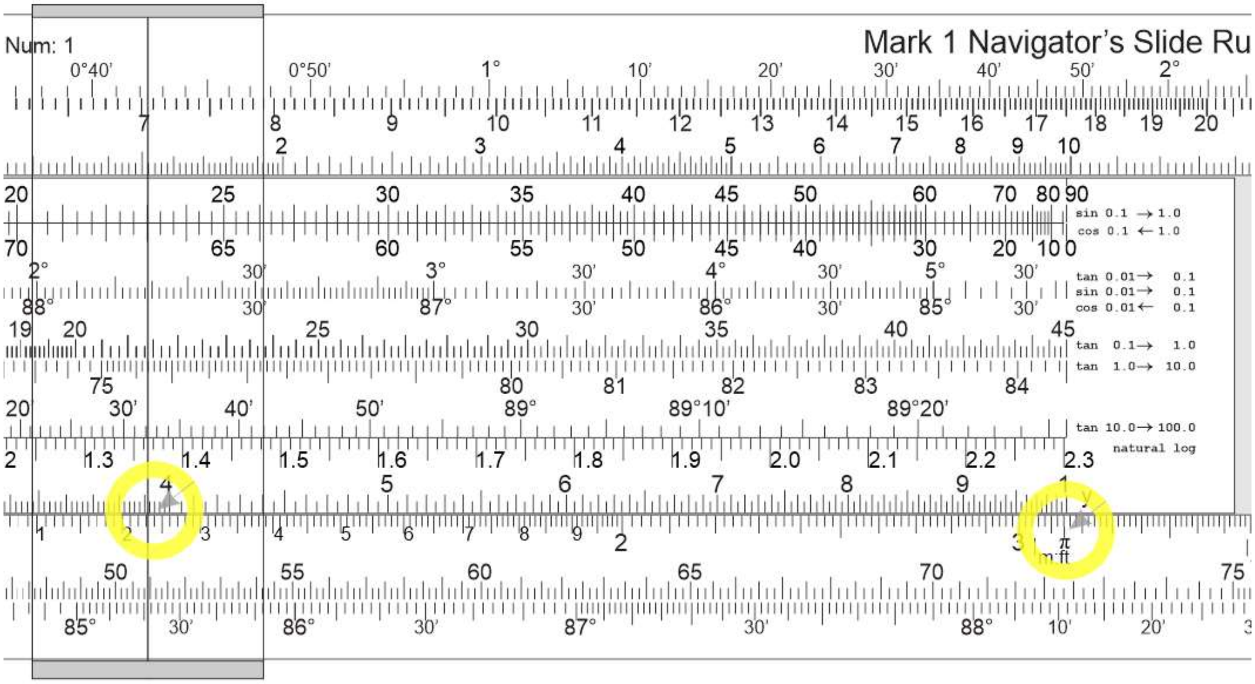
The reason both great circle and rhumb line calculations always yield their results in terms of an azimuth angle (notated as Z) rather than a regular true bearings (notated as Zn or the bearing of the true course) is that in both cases, you are solving a navigational triangle.

This being the case, it makes sense that you are never going to get any result to any equation where the direct answer is, say, 350°. Triangles, by definition, simply never have angles that size.

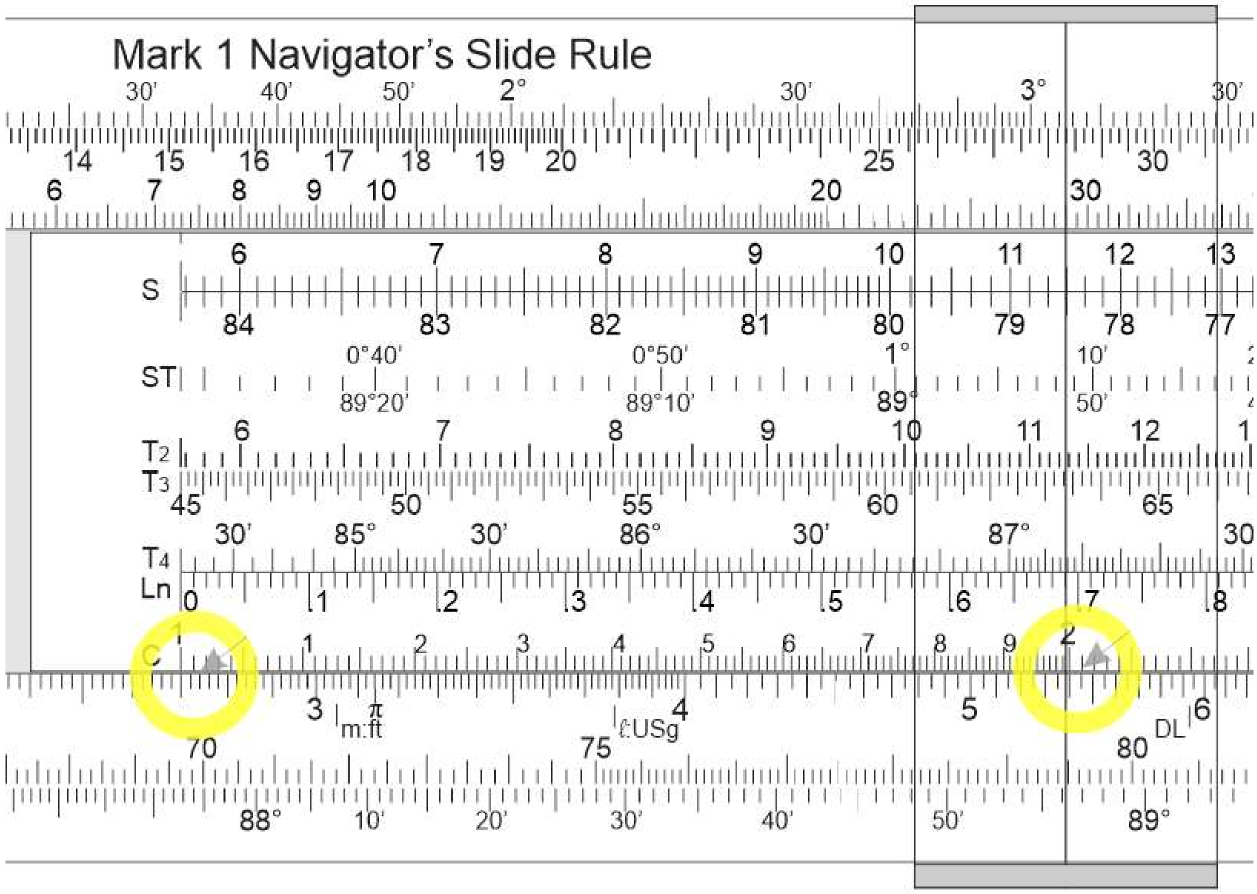
Steps 2 and 4: Conversion of degrees and minutes to decimal degrees. You don't need any more accuracy than to the nearest tenth of a minute. You can do this in your head. If you can't do it in your head, it is time to get in touch with your inner 10 year old, and review the "times-tables" that you learned in elementary school.[43] Do not use your slide rule for rudimentary calculations like these. Learn to think in tenths of a 60-minute degree

Step 8: π is marked on your slide rule just beyond 3.14. Align the right index over π. Move the slide over the top of 39.17 on the C-scale. Read the answer off the D-scale.

[43] I have a clear memory of being home from school in 5th grade with chicken pox, and having my parents set the task of memorizing the times-tables. If you are not already doing it, you need to easily visual 36 minutes of arc as 0.6°; 39 minutes of arc as 0.65°; 42 minutes of arc as 0.7°.



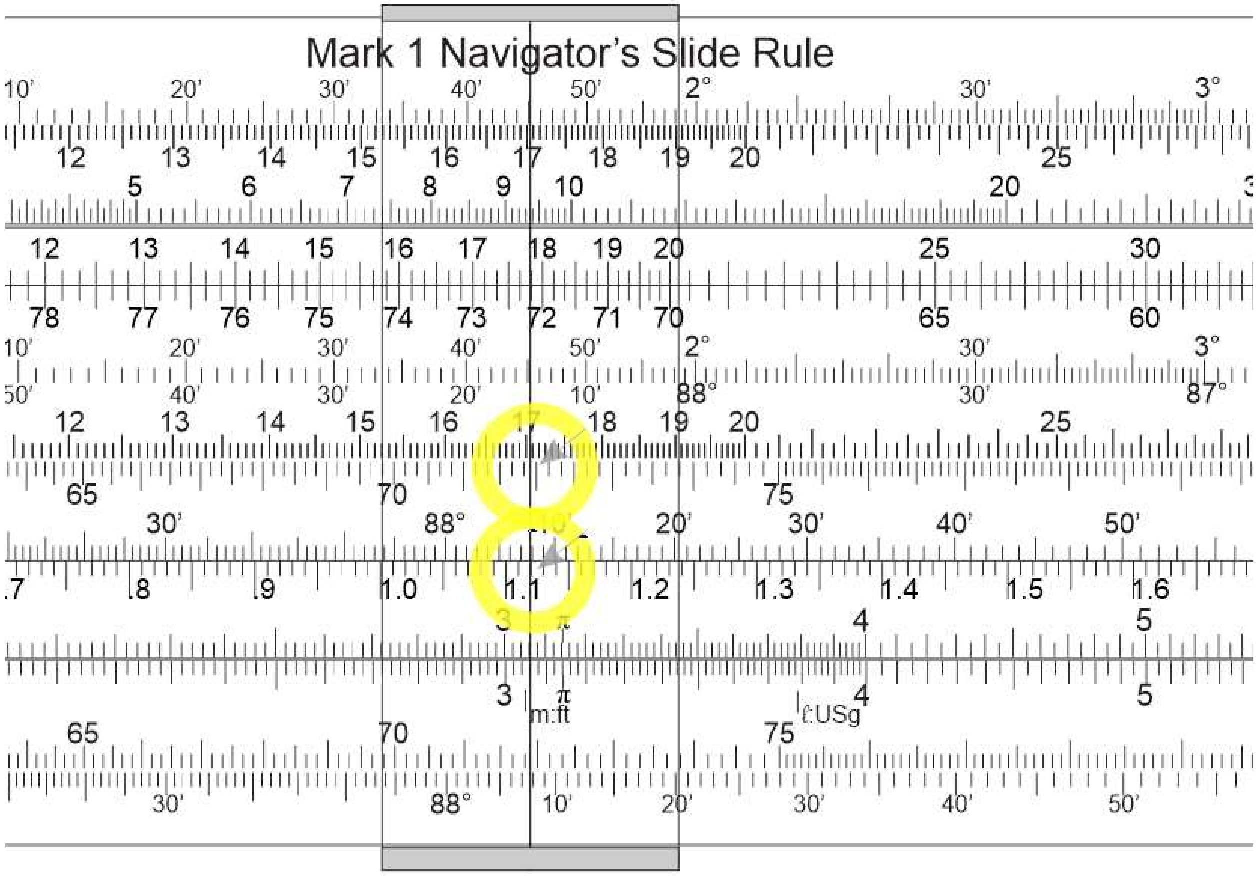
Step 9: Destination latitude, 53.9, divided by 2 looks like this. Read the answer on the D-scale below the left index.



Add the 27.92° you just got to 45° and you get 71.92°.

Step 10: You can convert 71.92 into degrees and minutes in your head as 71° 55' If you feel the need, you can multiply 60 * 0.92 using your slide rule and get the same answer.

Step 11: Move the slide over the top of 71° 55' on the T3 scale and then move your eye down to the Ln scale to get the value for this step.



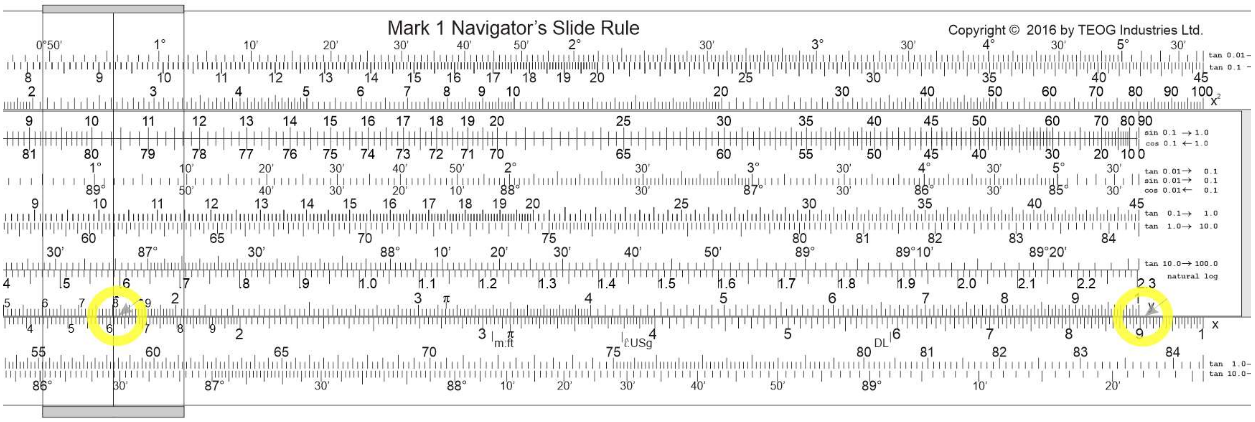
Steps 12 to 14: ...are performed using the same techniques as steps 9 to 11.

(my latitude ÷ 2) + 45 = (50.7° ÷ 2) + 45° = 25.35 + 45 = 70.35° = 70° 21'

Ln of tan(result from step 13) = 1.03

Step 15: Subtract line 14 from line 11 = 0.09.

Step 16: Multiply the result of step 15, 0.09, by 180 = 16.2.



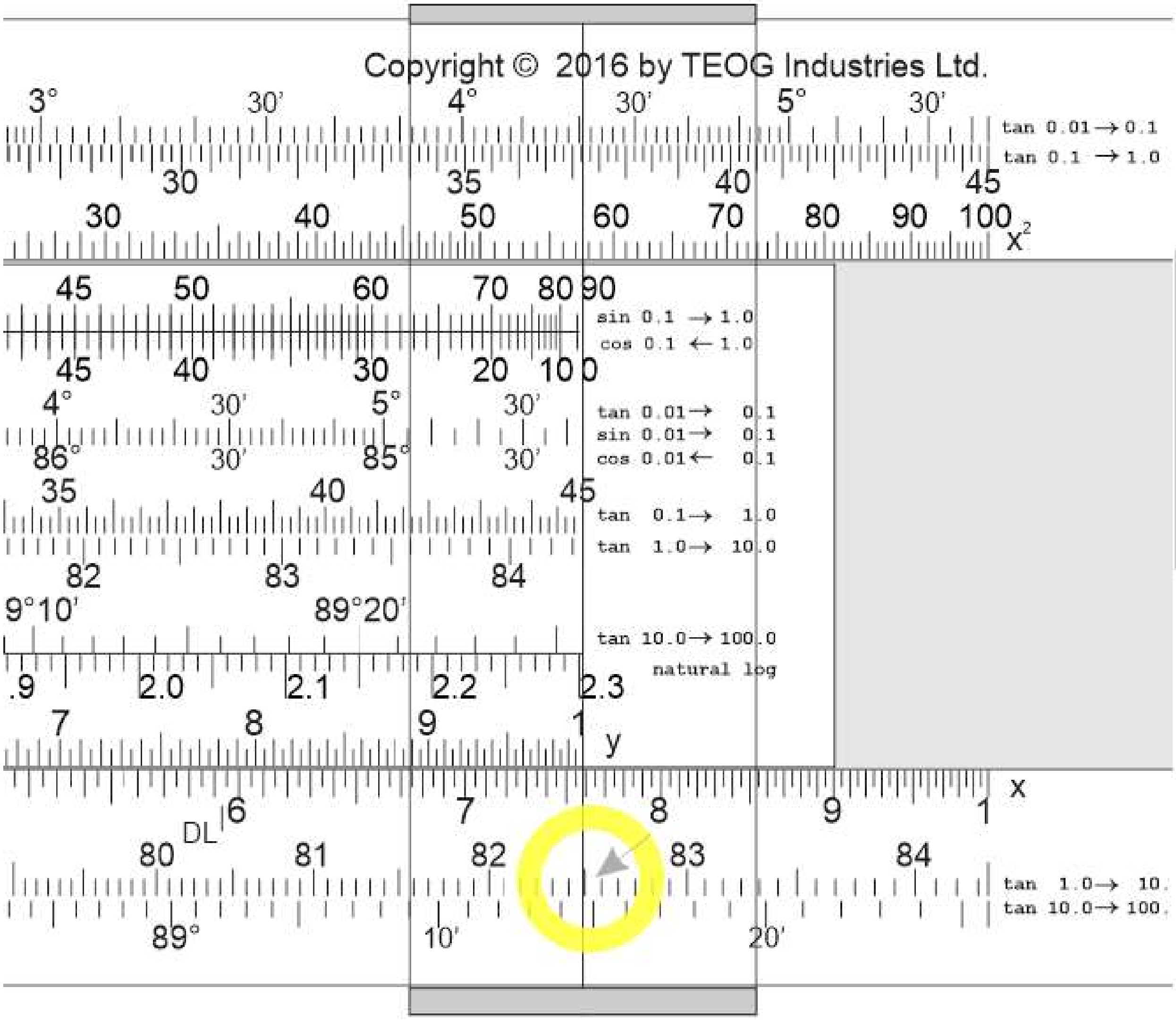
Step 17: Divide the number from step 8 by the number from step 16, or 122.9 ÷ 16.2 = 7.58.

Step 18: Read the arctan of 7.58 on the T5 scale = N 82° 29' W.

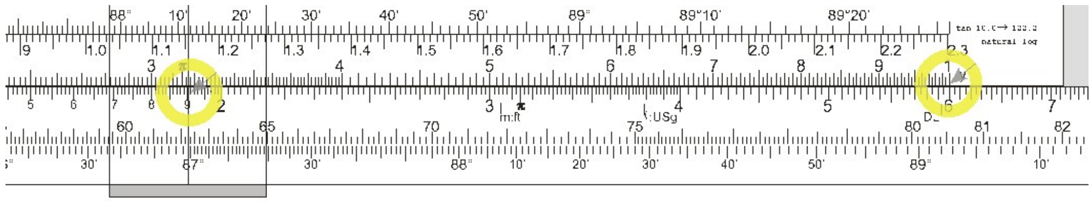
The azimuth values you get are prefixed with N if the destination is north of you; prefixed with S if the destination is south of you. So also, the suffix is W if the destination is to your west and E if it is to your east.

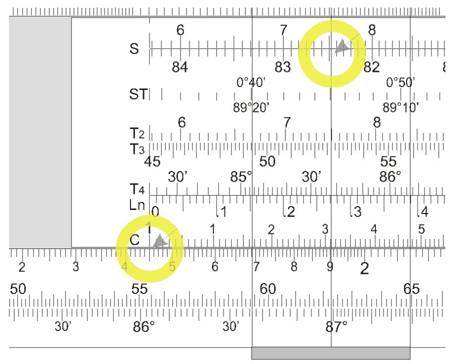
To convert your azimuth Z into an true bearing Zn (a.k.a. the course you want to steer), start from the north if the prefix is north, i.e. start from 360° and subtract the number of degrees and minutes indicated if the suffix is west. That is, you are going to move 82° away from north in a westerly direction.

If you had a Z of S 82° E, then Zn would be 98°. There is a diagram in section §44 above to help you relate Z values to Zn (or compass/true bearing) values.



Step 19: 60(ΔL) ÷ cos(RL course angle) = 60(3.17) ÷ cos(82° 29') = 190.1 ÷ cos(82° 29')
= 1,450 nm





If your SOG = 6.5 kn average = 156 miles per day, but beating into prevailing westerlies, your VMG would be closer to 78 nm per day. The difference between the rhumb line course from Port Hardy to Dutch Harbor at 1450 nm and the great circle at 1422 is 27 nm. This works out to a time savings of around 8 hours, all other things being equal. You are going to spend 18 days or so to travel this distance on a beat. If you want to navigate a great circle route, you will spend 4 or 5 minutes a day recalculating your optimal course for the sake of an 8 hour savings overall. You can decide if you would rather arrive in Dutch Harbor ASAP, at the cost of recalculating your route each day...or if you are willing to pay an 8 hour price in time and enjoy the simplicity of trying to make good a course of 82.5° and hold to it throughout.

The closer your ports are, the less sense it makes to go to the trouble of using a great circle route for vessel navigation.

However, if you were attempting to sail non-stop from Port Hardy to Tokyo non-stop, the rhumb line distance would be 4,126 nm and the great circle distance would be 3,898 nm. The great circle route would save you 228 nm in overall distance to be covered. If you persisted in making 78 nm/day against prevailing westerlies, that would translate into arriving 3 full days sooner.

So the further apart your ports are, the more sense it makes to sail great circle routes.

Great circle calculations are useful both for course planning for your vessel, but also useful for doing sight reduction if you are sighting a celestial object with a declination outside the range that Pub. 249 vol 2 and 3 work with. Rhumb line calculations are only used in vessel navigation.

§47. *Tacking Downwind*

When your destination is almost directly downwind of you,[44] you notice that if you point a bit higher, you go faster. You know that you are sailing a little further, but your instinct is that by sailing faster, though a bit further, you will arrive at your destination more quickly than if you sailed the more direct course.

This instinct tells you that there is a relationship between *distance* travelled, *speed*, and *time*. And indeed, with years of experience, a skipper will learn the optimal downwind tacking angle to use for his boat in different wind and sea conditions. But Arvel[45] Gentry published an important article in 1970[46] in which he outlined the math that will allow you to sail downwind efficiently without waiting all those years. He gives, if you will, the math that supports your instincts.[47]

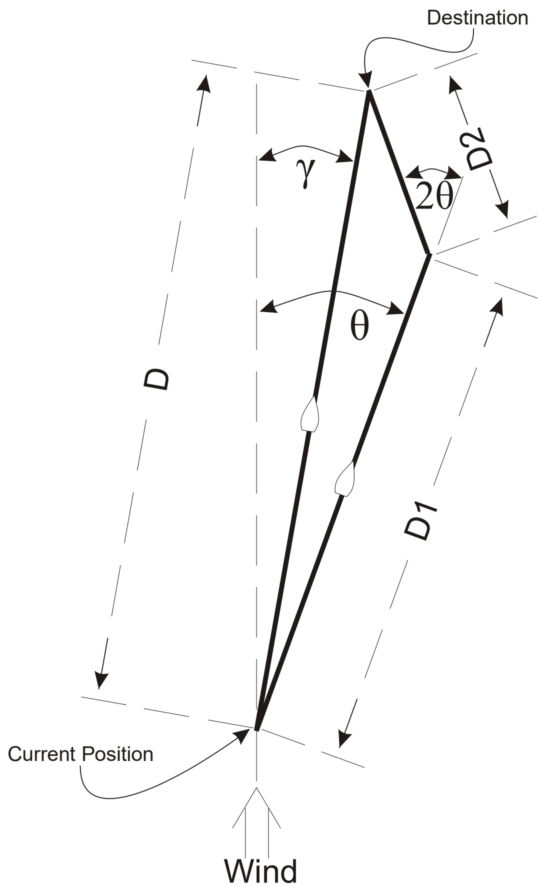
> Gentry, who died in 2015, published a number of articles over a period of years in which he brought his experience in aircraft design to bear on areas of interest to sailors.
>
> As of 05 March 2016, you can download a number of these articles[48] from http://arvelgentry.jimdo.com/ and http://tinyurl.com/AG-Articles .

Assume that the wind is coming from 205°. So if you are sailing at 25°, you are sailing dead downwind.

Assume the great circle route to your destination[49] is 35°. So then, if you are sailing 10° higher than dead downwind, you are precisely on course for your destination. But will you get there faster if you sail 20° higher, or even 30° higher than dead downwind?

---

[44] It is seldom *exactly* downwind. At the end of this section, I will explain how this math works to take care of this rare scenario as well.
[45] ...an old Welsh name.
[46] He was, at the time, Chief of Applied Research at Douglas Aircraft. By 1986, he was head of Boeing's aerodynamics computing department, and in his spare time was helping to design a winged keel for one of the America's Cup challengers.
[47] http://tinyurl.com/TackingDownwind (retrieved 05 March 2016)
[48] …with the blessing of his daughter, who is more interested in seeing her father's legacy carried on than in protecting copyrights. http://tinyurl.com/KimGentry (retrieved 05 March 2016)
[49] or the rhumb line course, if you are doing coastal sailing.



γ = angle between line to destination and dead downwind direction

θ = downwind tacking angle

D = distance to mark without tacking

Dθ = D1 + D2, i.e. distance to destination with downwind tacking (further than D but potentially faster to complete)

2θ = course to follow after gybe (= twice θ)

Sγ = speed you make on direct course to destination, at angle γ to dead downwind

Sθ = speed you make on course θ, pointing higher from dead downwind than γ

Stating the question more generally, you know γ. You want to know the optimum angle for θ which will get you to your destination the very fastest. You don't care if Dθ is further than D. You care about getting home quickly.

The complication in this is that the optimum answer will be different for different hull shapes and sail combinations, and also for different sea states and wind speeds. So then, you must begin by observing the actual speed you make while following different courses. That is to say, it will take several minutes to collect the data before you can calculate your optimum course.

Then, you need a "distance factor" (DF) and a "speed factor" (SF) that you can relate a "time factor" (TF). These are:

$$\mathrm{DF} = \frac{\cos(\gamma)}{\cos(\theta)} \qquad \mathrm{SF} = \frac{S\theta}{S\gamma} \qquad \mathrm{TF} = \frac{\mathrm{DF}}{\mathrm{SF}}$$

The time factor represents the fraction of time it will take to sail on the given course θ compared to the baseline time it will take to sail on course γ. If the TF = 0.750, then it means that sailing course θ will take 75% of the time it would take if you sailed on course γ. Another way of saying is is that course θ is 25% faster under these conditions.

Example with a TEOG 40 foot sloop, where γ is 10° higher than dead downwind.

That is, if the wind is coming from due south, 180°, dead downwind would be 0°. Pointing 10° closer to the wind than 0° would be sailing at 10° (or giving equal answers, sailing at 350° - but we are not interested in true bearings; only bearings relative to dead downwind).

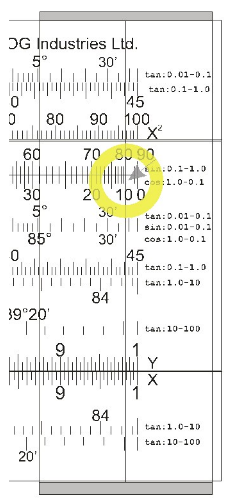
Conditions: 4 knot wind; small waves with glassy crests; no breaking waves.

Course to Dest. relative to dead downwind γ	Course sailed relative to dead downwind θ	Distance Factor DF $\frac{\cos(\gamma)}{\cos(\theta)}$	Speed on course θ $S\theta$	Speed Factor SF $\frac{S\theta}{S\gamma}$	Time Factor TF $\frac{DF}{SF}$	Ideal Course
10	10	1.000	2.0 (i.e. $S\gamma$)	1.000	1.000	
	20	1.048	2.3	1.150	0.911	
	30	1.137	2.9	1.450	0.784	
	40	1.286	3.4	1.700	0.756	★
	50	1.532	3.8	1.900	0.806	

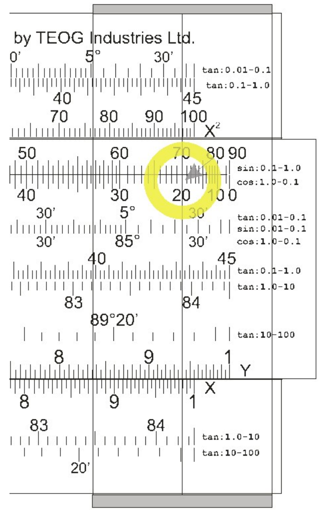
Let's unpack this. If you are sailing directly at your destination, pointing 10° higher than dead downwind, your speed is 2.2 knots. Of course, at that angle $\cos(\gamma)/\cos(\theta) = \cos(10°)/\cos(10°) = 1$. You will find that your speed factor and time factor are both 1. This is your baseline result. You want to see if you can do better than this.

The DF for 20° is cos(10°) ÷ cos(20°) = 1.048

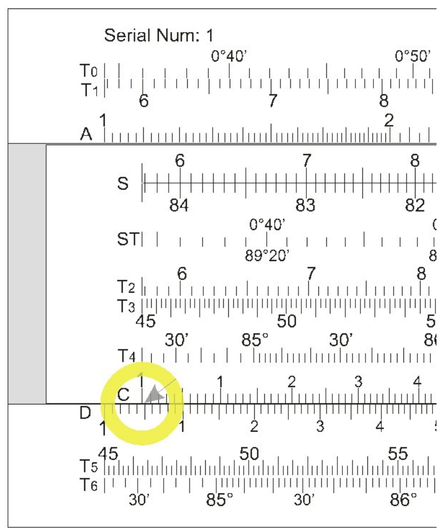
Position your cursor over cos(10°).



Now move the hash mark for cos(20°) underneath the cursor, to do a division.



Read the answer on the D scale, under the left index on the C scale.



Do this for the other values of θ you want to check: 30°, 40°, and 50°.

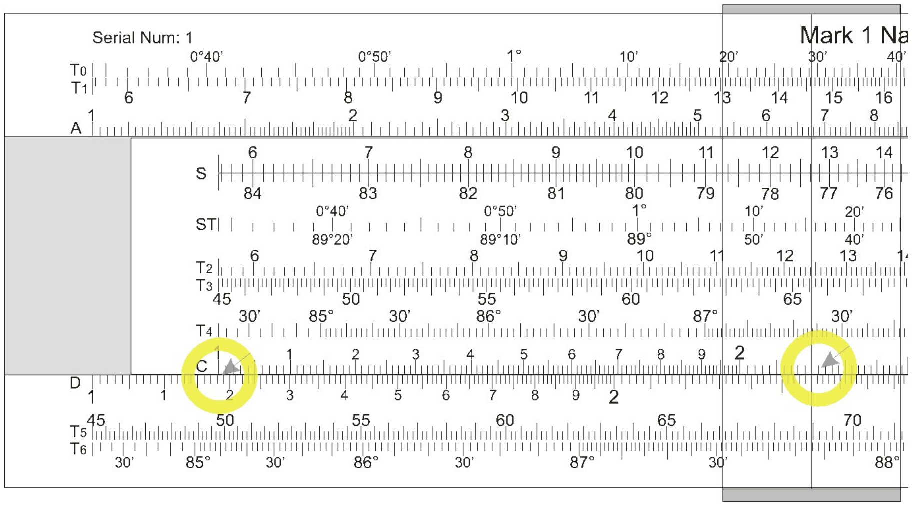
Now, settle your craft in for a few minutes on each course, starting at 10° (i.e. γ, pointing directly at your destination), 20°, 30°, 40°, and 50°. You will be using the masthead wind vane and your instruments to ensure that you are on the proper course. In each case, allow a few minutes to pass for your speed to settle down. In 4 knots of wind, your speeds should be quite consistent.

As you move into higher winds, and bigger waves, your speed will vary more from moment to moment.[50] Do your best to get an average speed to the nearest tenth of a knot.

Now that you have the first four columns of your chart for 4 knots of wind filled in, you can calculate SF. This is a straightforward division. When θ = 20° and your speed is 2.6 kn,

[50] You will slow down as a big wave smashes into your bow.

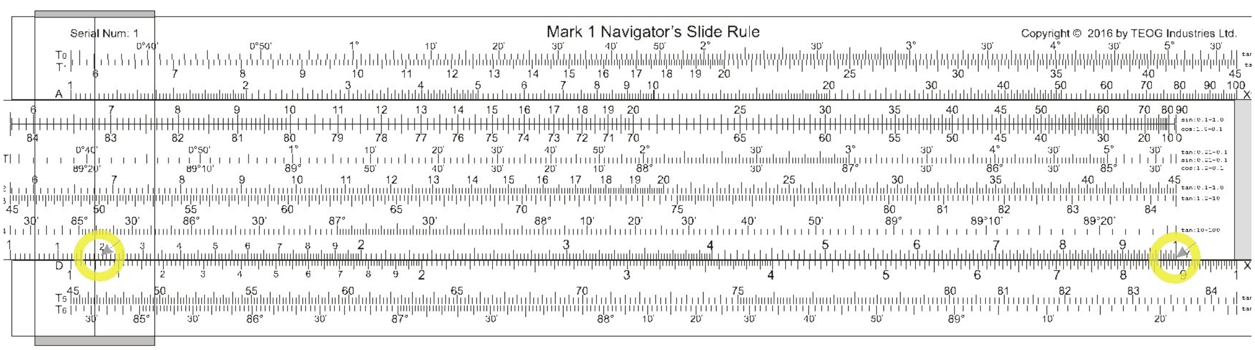
then SF = 2.6 kn/2.2 kn. Position 2.2 on C over 2.6 on D, and read the answer on D under the left index of C = 1.182.



Carry this on to calculate the speed factor values for the other course values you want to evaluate.

Now, fill in the time factor column, where TF = DF/SF. When θ = 20°, this is 1.048/1.182 = 0.887.

Position the cursor on 1.048 on the D scale, and position 1.182 on the C scale under the cursor, and read the answer under the right index of the C scale.



The smaller the time factor value, the quicker you will arrive at your destination. In this case, sailing 40° higher than dead downwind will get you to your destination in 74.5% the time it would take you if you sailed directly for your destination.

You should understand how to work the slide rule on this problem, but you may be struggling to wrap your head around this whole concept of calculating tacking downwind angles. It may help to use this same TEOG 40 example at some different wind speeds.

Conditions: 8 knot wind; scattered whitecaps.

Course to Dest. relative to dead downwind γ	Course sailed relative to dead downwind θ	Distance Factor DF $\frac{\cos(\gamma)}{\cos(\theta)}$	Speed on course θ $S\theta$	Speed Factor SF $\frac{S\theta}{S\gamma}$	Time Factor TF $\frac{DF}{SF}$	Ideal Course
10	10	1.000	4.3 (i.e. $S\gamma$)	1.000	1.000	
	20	1.048	5.3	1.233	0.850	
	30	1.137	6.0	1.395	0.815	★
	40	1.286	6.7	1.558	0.825	
	50	1.532	7.0	1.628	0.941	

Conditions: 12 knot wind; numerous whitecaps.

Course to Dest. relative to dead downwind γ	Course sailed relative to dead downwind θ	Distance Factor DF $\frac{\cos(\gamma)}{\cos(\theta)}$	Speed on course θ $S\theta$	Speed Factor SF $\frac{S\theta}{S\gamma}$	Time Factor TF $\frac{DF}{SF}$	Ideal Course
10	10	1.000	6.3 (i.e. $S\gamma$)	1.000	1.000	
	20	1.048	7.3	1.233	0.904	★
	30	1.137	7.8	1.395	0.918	
	40	1.286	8.2	1.558	0.988	
	50	1.532	8.2	1.628	1.077	

Conditions: 16 knot wind; 7 foot waves, ubiquitous whitecaps, some spray.

Course to Dest. relative to dead downwind γ	Course sailed relative to dead downwind θ	Distance Factor DF $\frac{\cos(\gamma)}{\cos(\theta)}$	Speed on course θ $S\theta$	Speed Factor SF $\frac{S\theta}{S\gamma}$	Time Factor TF $\frac{DF}{SF}$	Ideal Course
10	10	1.000	7.9 (i.e. $S\gamma$)	1.000	1.000	★
	20	1.048	8.0	1.013	1.035	
	30	1.137	8.2	1.038	1.096	
	40	1.286	8.2	1.038	1.239	
	50	1.532	8.2	1.038	1.476	

What you are observing here is that when winds are light, and you are nowhere near your maximum hull speed, pointing higher than your rhumb-line course by 30° or 40° could be a good idea.

But as your boatspeed goes up, progressively closer and closer to your maximum hullspeed, then your ideal course moves closer to pointing directly at your destination. With this particular TEOG 40, once the wind gets to 16 knots, going faster through the water really isn't an option. 8.2 knots is as fast as you are going to go.[51] Your best plan is to point directly at your destination.

Conditions: 24 knot wind; 12 foot waves, the boat is surfing with speeds up to 12 knots.

All these equations go right out the window when you start to surf. Hang onto your hat and have fun!

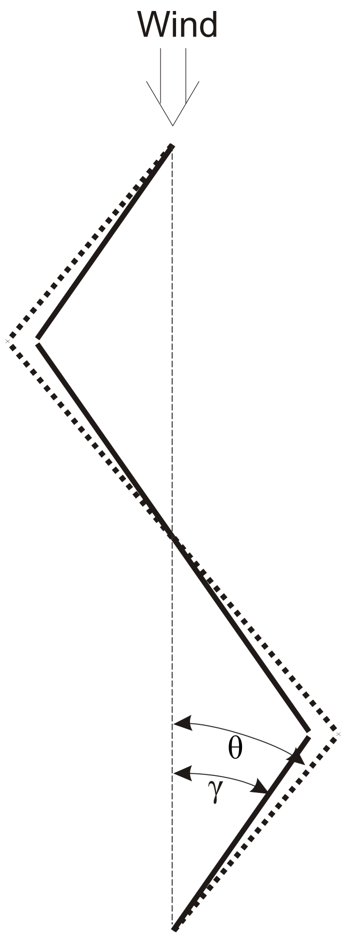
Use the same equations on those occasions when your destination is dead downwind. Just use $\gamma = 0°$, and work the rest of your table as normal.

Inshore racing sailors would do well to start a notebook and gather data for a wide range of wind speeds and sea conditions, the better to know how their boat and sails perform.

[51] If you think that this is not quite up to what the equation for maximum hull speed would suggest, you need to recall that the hull speed equation deals in LWL (length at water line) and not LOA (length over all). The TEOG 40 has a LOA of 40 feet. Its LWL is shorter.

§48. *Tacking Upwind*

Selecting an optimal course while tacking upwind is very much the same problem, mathematically, as tacking downwind.



γ = angle off the wind - pointing as high as possible (e.g. 30°)

θ = actual angle off the wind pointing lower

D = total distance traveled when pointing as high as possible

$D\theta$ = total distance travelled when pointing at angle θ

$S\gamma$ = speed you make pointing as high as possible

$S\theta$ = speed you make while pointing lower, at angle θ from the wind

In sailing on the wind, your speed drops off rapidly whenever you sail higher than 35° or so, depending on the boat. You accelerate as you sail lower, but of course you travel further. Your objective is to find a course that is optimized for speed in the particular sailing conditions in which you find yourself.

Once again, you need a "distance factor" (DF) and a "speed factor" (SF) that you can relate a "time factor" (TF). These are:

$$DF = \frac{\cos(\gamma)}{\cos(\theta)} \qquad SF = \frac{S\theta}{S\gamma} \qquad TF = \frac{DF}{SF}$$

Course off wind when pointing as high as possible γ	Course off wind, pointing lower θ	Distance Factor DF $\cos(\gamma)/\cos(\theta)$	Speed on course θ $S\theta$	Speed Factor SF $S\theta/S\gamma$	Time Factor TF DF/SF	Ideal Course
30°	30°	1.000	5.8	1.000	1.000	
	32°	1.021	6.3	1.086	0.940	
	34°	1.045	6.7	1.155	0.904	
	36°	1.070	7.2	1.241	0.862	
	38°	1.099	7.4	1.276	0.861	★
	40°	1.131	7.6	1.310	0.863	

You will observe that when beating upwind, you will likely be evaluating course variations in 2° increments. Sailing off the wind, one is more likely to evaluate course variations in 10° increments.

§288.1 The Role of Polar Diagrams

If you have a polar diagram, then by all means make use of it! The boat I sailed on a month ago was a PY26, made 40 years ago...but still in superb shape. I found a copy of the original owner's manual, but it appears the manufacturer never created a polar diagram for this vessel.[52] Further, the old, baggy sails had been replaced by a new set...but a set that was created for a different kind of vessel. The jib had a shorter luff and the main a shorter foot.

In this kind of situation, the boat owner will need to collect data to create his own polar diagram.

§49. Compass Delta

Creating a compass deviation card for a vessel is something everybody talks about but almost nobody does.[53] Let us suppose that you are among this majority who has never

[52] Was any manufacturer doing this 40 years ago?
[53] In 2016, I finally saw my very first actual compass deviation card, on a small aircraft that I rode in from Vancouver to Maple Bay, BC. Apparently, making such a card is a federal requirement for this type of aircraft.

measured your deviation, and that while you are sailing from Dutch Harbor to Tokyo, you decide it is a good time to begin creating a deviation card for yourself.[54]

The only true and utterly reliable way[55] to determine the magnetic delta (variation + deviation) of your steering compass,[56] at your current location and on your current course, is by comparing your compass reading to the azimuth to a celestial object. You can use your Nautical Almanac plus Pub. 249 to determine the azimuth of any of these "lighthouses in the sky", of course. But there is an easier, quicker way.

You already know that taking a meridian sight at local noon gives you a reliable measure of your latitude. Part of what makes this method useful is that although the sun is moving quickly from east to west at noon, its altitude is changing very slowly indeed. So you can get a very reliable sextant altitude.

By the same token, near sunrise or sunset, the sun's altitude is changing very rapidly, but its azimuth change from east to west is very slow. So you can get a very reliable read on your compass.

When the center of the sun is on the celestial horizon – which, due to refraction is when the lower limb of the sun is 2/3 of its diameter above the visible horizon[57] – you can calculate the azimuth with the equation:

Sin(Azimuth) = sin(declination) ÷ cos(latitude)

You don't need to include anything in this equation about the altitude of the sun, or any sort of corrections. If the center of the sun is on the celestial horizon, you have already eliminated the effect of refraction. Hence the true altitude at that moment = 0°.

---

[54] You don't need to collect all your data at once. Also note that there may be a heeling error, as the relative positions of the engine and compass change. Compass deviation issues can lead you into the deep end of the pool, so to speak. But keep in mind that if you had no deviation card whatsoever last week, what you are doing today is a genuine improvement, even if it is not perfect.
[55] http://www.pbo.co.uk/expert-advice/how-to-swing-a-compass-17845 gives a method that assumes you can find a place to stand in the boat where the hand-bearing compass is unaffected by any deviation. Try this out! If this works, it could take less time to complete than the celestial method outlined above.
[56] I spent a couple of weeks on a vessel that had two conventional compasses in the cockpit, plus a digital compass. No two of these compasses agreed with each other, and no single compass agreed with the GPS. The maximum differences between reported courses amounted to 35° on certain headings.
GPS is of some value in determining compass deviation, of course. But if we are sailing on salt water, then most of us, most of the time, are sailing across a current of not-precisely-known direction and not-precisely-known speed. GPS tells you how the boat is moving without regard for which way the bow is pointing. To really dial in your compass deviation, there is no substitute for a celestial object.
Having done this, you can use the difference between your compass heading and GPS heading to infer something about any current you may be in. (Section §30 will give you a starting place to work from.) Particularly in the open ocean, this may be the only way to get useful current data.
[57] Bowditch (2002). p. 273.

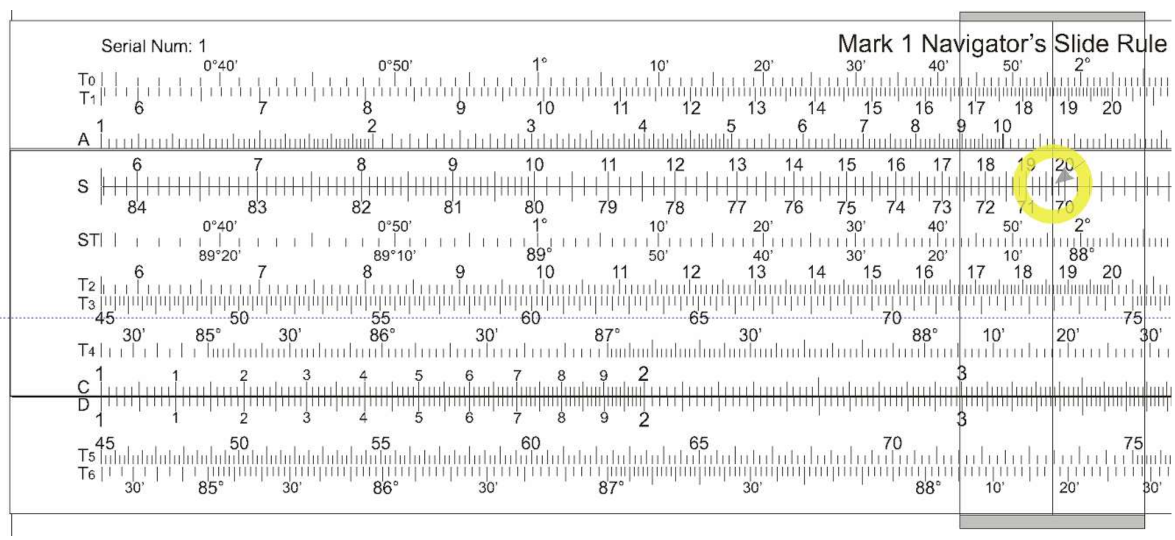
With the azimuth from this equation, turn your sextant on its side to measure the angle from the center of the sun to the bow (or center of the stern) of the boat, and determine the delta (i.e. the difference) between your steering compass and true bearings. Make this a part of your morning/evening routine, along with taking sextant observations during nautical twilight, and you can dial in your compass delta to a fare-thee-well. Once you get back home, you can turn this data into a proper deviation card.

To make this work, you need your own latitude. You also need, from the Nautical Almanac, the declination of the sun.

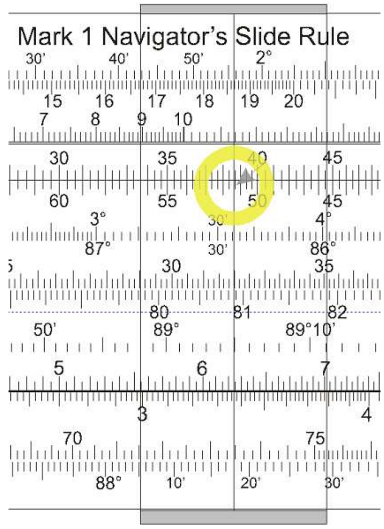
For example: The DR position of your vessel is 51° 24.6' N, 179° 00.0' E. Your steering compass indicates your course as being 252°. The declination of the sun is 19° 40.4' N. You want the azimuth of the sun.

Rules: All values are treated as positive, whether or not latitude and declination are of contrary name. The azimuth is given the prefix of E if the sun is rising; W if it is setting. It is given the suffix N if the declination is north; S if the declination is south.

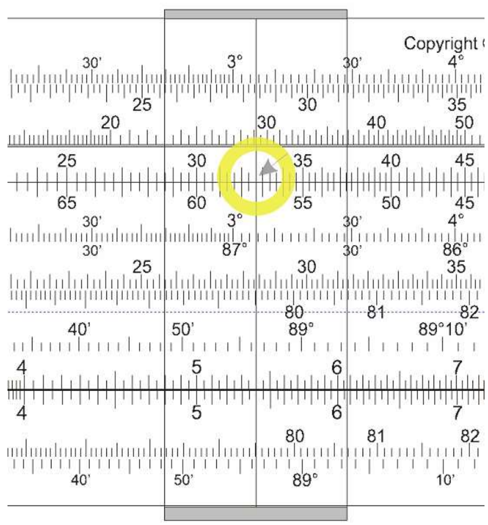
Center the slide, then position the cursor over the top of sin(19° 40').



Position cos(51° 25') beneath the cursor to divide.



Position the cursor over the top of the right index, which yields the answer to this problem in division, then center the slide. Read the azimuth of the sun on the top of the S scale: W 32° 40' N.



Azimuth = W 32° 40' N, which is the same as…

Azimuth = 270° + 32° 40', which is the same as…

Azimuth = 302° 40'…which we will round to 303° [58]

If you had a compass delta of 0° then the bearing difference from the center of the sun to your bow (which is on course 252°) would be 303° - 252° = 51°.

---

[58] Typically, you cannot read a binnacle compass with any more accuracy than ± 1° of so. Hence, you will round your numbers to the nearest degree. If your steering compass' design is such that you can read it only to the nearest 5°, then just do the best you can.

But when you use your sextant to measure from the bow to the center of the sun, you get an angle of 53°. That is to say, your compass delta is 2° west. Write this down in your log, together with the DR position of your vessel.

Then once you get back on dry land, go to this web address:

http://www.ngdc.noaa.gov/geomag-web/

…enter your DR position of 51° 24.6' N, 179° 00.0' E, and you will discover that at that location on the face of the earth, the magnetic variation is 3.05° east[59] which we will round to 3° east.

```
Compass Δ:        2° west
Variation:        3° east
                 --------
Deviation:        5° west
```

Go through this same process as your vessel is on different magnetic courses over the course of your cruise, and you will eventually build up the data for a useful deviation card. See http://www.cockpitcards.co.uk/page15.htm.

While using the sun is the classic way to determine the magnetic variation at your DR, you can also use the moon: measure with your sextant from the moon to the bow of the boat when the *upper* limb of the moon is just touching the visible horizon.

If you wish to use a star or planet to check your magnetic variation, then do your sextant observation when the object is just a little bit more than one sun diameter above the visible horizon.

Looking back over what we have done, once we knew the magnetic variation, we knew everything but the deviation. Once we have worked out a full deviation card, we can of course then solve for the variation wherever we are in the earth.

§50. *Compass Delta and the Sun Compass*

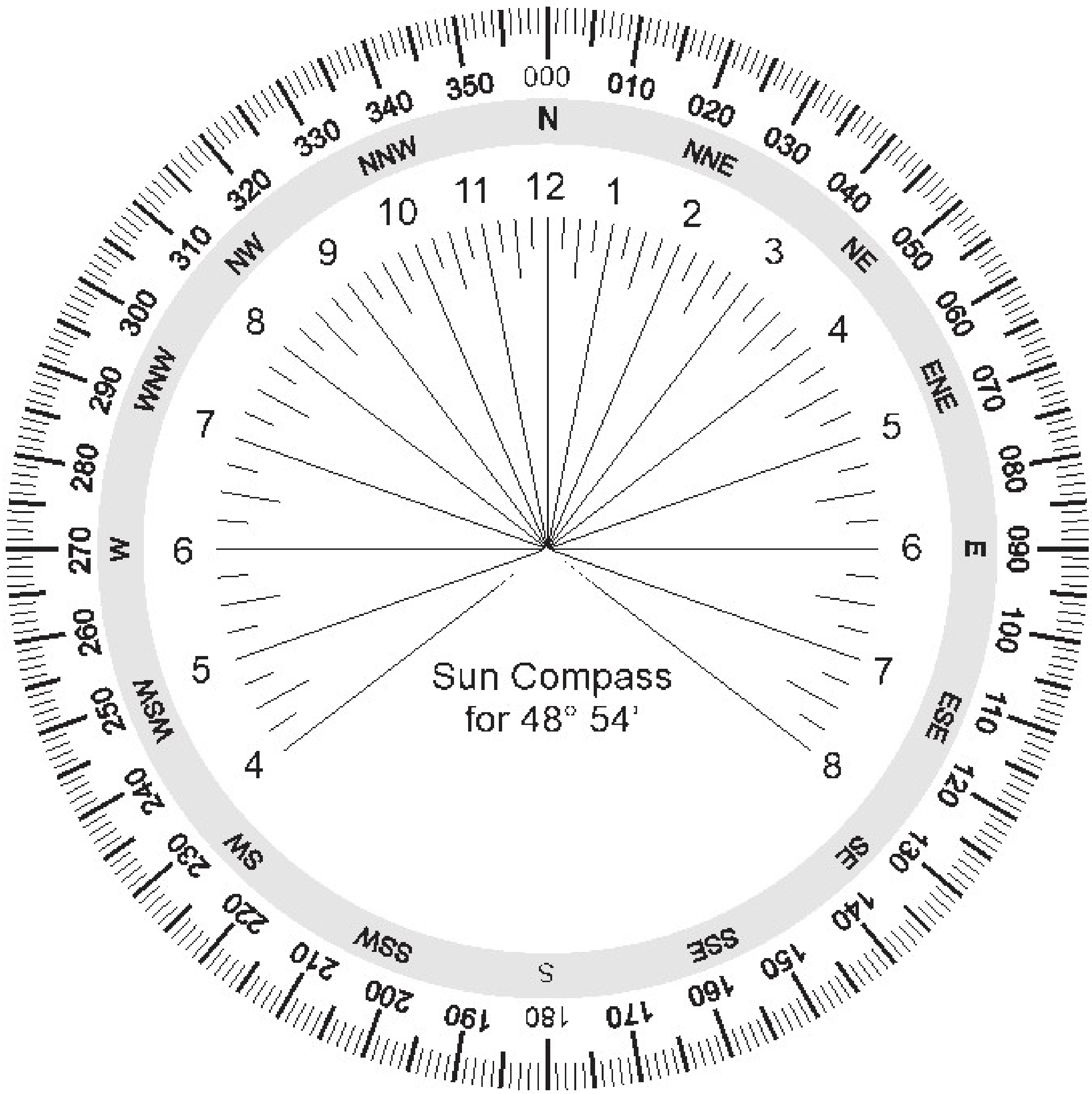
There is one technique for determining compass delta which, once you have it built, requires no further mathematics and very little time to use: the sun compass.

---

[59] On both US and Canadian government websites, the scientists that calculate earth magnetism talk in terms of "magnetic declination". The hydrographic offices that publish nautical charts describe the same phenomenon as "magnetic variation."

Normally, if you know what direction north and south are, you align your sundial and it tells you the time.

A sun compass and a sundial are the same basic instrument. The difference is that with a sun compass, you know the time, but do NOT know directions. Make correction for the equation of time, plus your longitude, to determine your local solar time (LST). Turn the sun compass until the time is matches LST, and you can observe true directions.

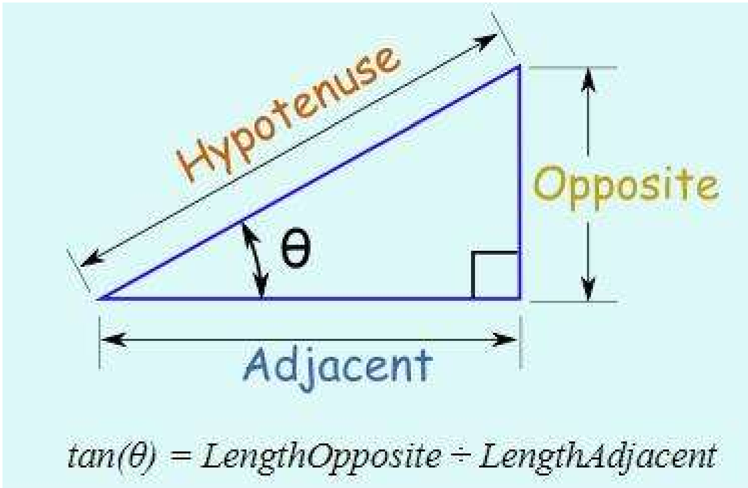


To build this, you will need a vector drawing tool like CorelDraw or Adobe Illustrator (or if you prefer open source, InkScape). When I did this, I laid out 360° of markers in a pattern the same diameter as a DVD. Then, either using a website such as http://www.anycalculator.com/horizontalsundial.htm, or using the equation $\theta = \arctan(\sin(L) * \tan(15° * t))$, where:

- θ = the angle between a given hour-line and the noon hour line
- L = your latitude
- t = the number of hours before or after noon

...lay out your sun compass' hour lines.

The gnomon needs to be at the same angle to the face of the sundial as your latitude. You can calculate the proper length of your gnomon with this equation:

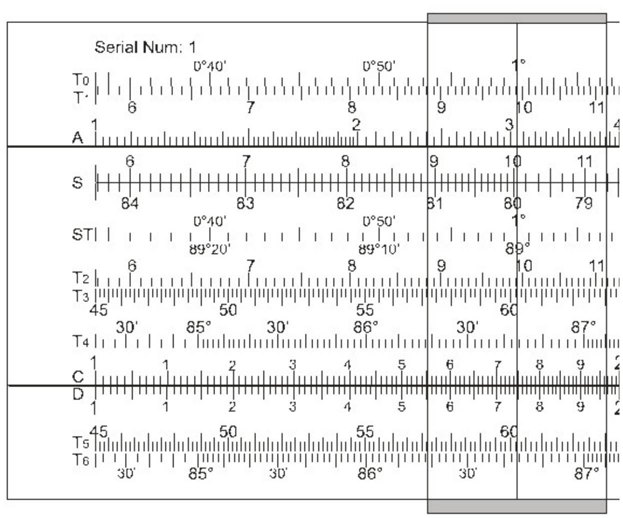


As long as you are sailing near to the latitude for which your sun compass was designed, you can use it to determine the delta of your yacht's steering compass. However, since you are normally in your home waters when you try to come up with your deviation card, this is not a significant limitation.

§51. *Trigonometry and Mental Math*

Generally, when using a slide rule, you do some quick-and-dirty calculation to come up with the general scope of the solution to your problem. This lets you know where to put the decimal place.

But if you have no idea the scope of an answer, you have a problem. After all, if your slide is in this position:



…then when the answer is a tangent value, it could be:

- 1° 00' from the T_0 scale

- 9° 56’ from the T_1 scale
- 60° 12’ from the T_5 scale, or…
- 86° 42’ from the T_6 scale.

As you are aware, the Mark 1 lets you work with trigonometric values quickly and directly, without needing to concern yourself with the numeric values into which they translate. But if you get confused about the scope of your answer, then a bit more understanding will be helpful.

Sine

Using the ST scale from 0° 34' to 5° 44' = sine values from 0.01 to 0.1.

Using the S scale from 5° 44' to 90° 00' = sine values from 0.1 to 1.

Cosine

Using the S scale from 0° 00' to 84° 16' = cosine values from 1 to 0.1.

Using the ST scale from 84° 16' to 89° 26' = cosine values from 0.1 to 0.01.

Tangent

Using the T_0/ST scales[60] from 0° 34' to 5° 44' = tangent values from 0.01 to 0.1[61]

Using the T_1/T_2 scales[62] from 5° 44' to 45° 00 = tangent values from 0.1 to 1

Using the T_3/T_5 scales from 45° 00' to 84° 17' = tangent values from 1 to 10

Using the T_4/T_6 scales from 84° 17' to 89° 26' = tangent values from 10 to 100

Putting It All Together

If you see an equation that is:

$\sin(32°) \times \cos(70°) = \tan(\theta)$

…and you simply have no foggy clue as to even the general value that θ represents, then restate the equation in terms of the trig values.

sin(32°) × cos(70°) is an equivalent equation to **0.530 × 0.342**.

---

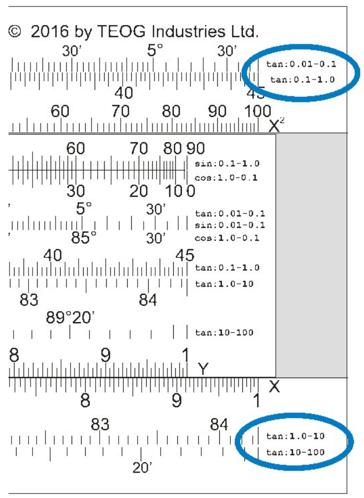
[60] The top half values of the ST scale are identical to the T_0 scale values. I could have as easily named T_0 as ST_0 or some other similar designation.
However named, the scale needed to be up on the body to facilitate easy calculation of great circle routes.

[61] Within the limits of slide rule accuracy, sine and tangent values are identical for any angles less than 5° 44'.

[62] Because this ruler has been optimized to make the Bygrave equations for sight reduction and great circle calculations easy and quick, all of the possible tangent scales appear both on the slide and on the body.

So you work out in your head that 0.530 ≅ ½. You figure that ½ of 0.34 is 0.17. You know that all tangent values from 0.1 to 1 lie along the T2/T5 scales. So if tan(θ) is your answer, and it is close to 0.17, then θ will be 10° 16'.

As a memory aid, the range of each trig scale is documented on the rule at the right edge of the scale.



§52. Small Cosine Values

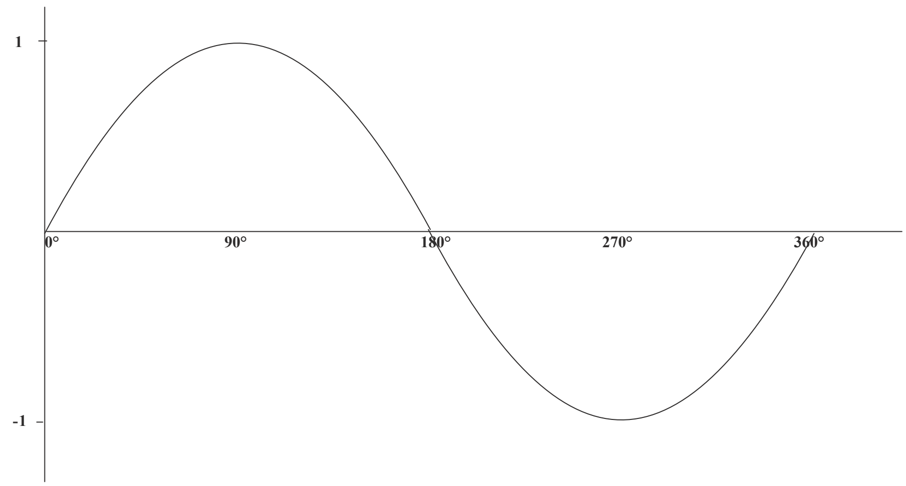
Cosine values between 0° and 10° are difficult to read, as the S scale is so compact, and the spaces between each degree are not consistent. That is, the cosine function is extremely non-linear in this range. (Of course, the same thing applies to sine values between 80° and 90°.)

It can be a help to use the natural log values from the Ln scale to set the cursor for small cosine values, since the Ln scale is linear and interpolation is a good bit easier. Still, once you get down to the difference between cos(1°) and cos(2°), it is impossible to manage accurately on a slide rule.

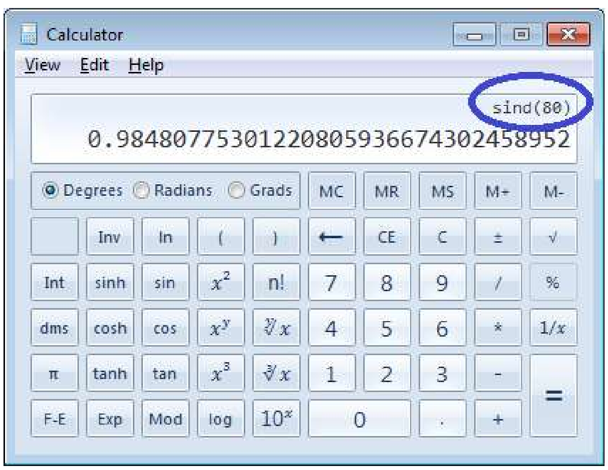
Cosine	Sine	Ln
9°	81°	2.290
8°	82°	2.293
7°	83°	2.295
6°	84°	2.297
5°	85°	2.299
4°	86°	2.300
3°	87°	2.301
2°	88°	2.302
1°	89°	2.302

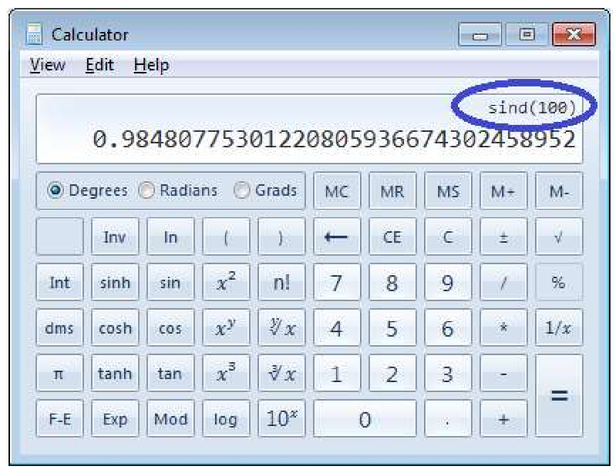
§53. Angles Larger Than 90°

A slide rule deals in angles from roughly than 0° through to 90°. But if you are dealing in a meridian angle larger than 90° (as with a great circle calculation from San Francisco to Tokyo), you need to be able to work with angles larger than 90°. You can do this if you remember what a "sine curve" looks like.



Sine values, for instance, climb as you go from 0° to 90°, then fall as you go from 90° to 180°. You can confirm with an electronic calculator that the sine of 80° (which is 10° less than 90°) is exactly the same as the sine of 100° (which is 10° more than 90°).





Visual Methods

§54. Calculating Intermediate Tide Heights

It is possible to make surprisingly accurate tide predictions using the scale of inches on the backside of the Mark 1 Navigator's Slide Rule, a pencil, and a piece of paper.

Even a quick-and-dirty graph may be sufficient[63] to calculate how much anchor rode you need to veer in order to sleep soundly that night.

Step 1: From the tide book, note the times and heights of the low and high tides during the time period that concerns you.

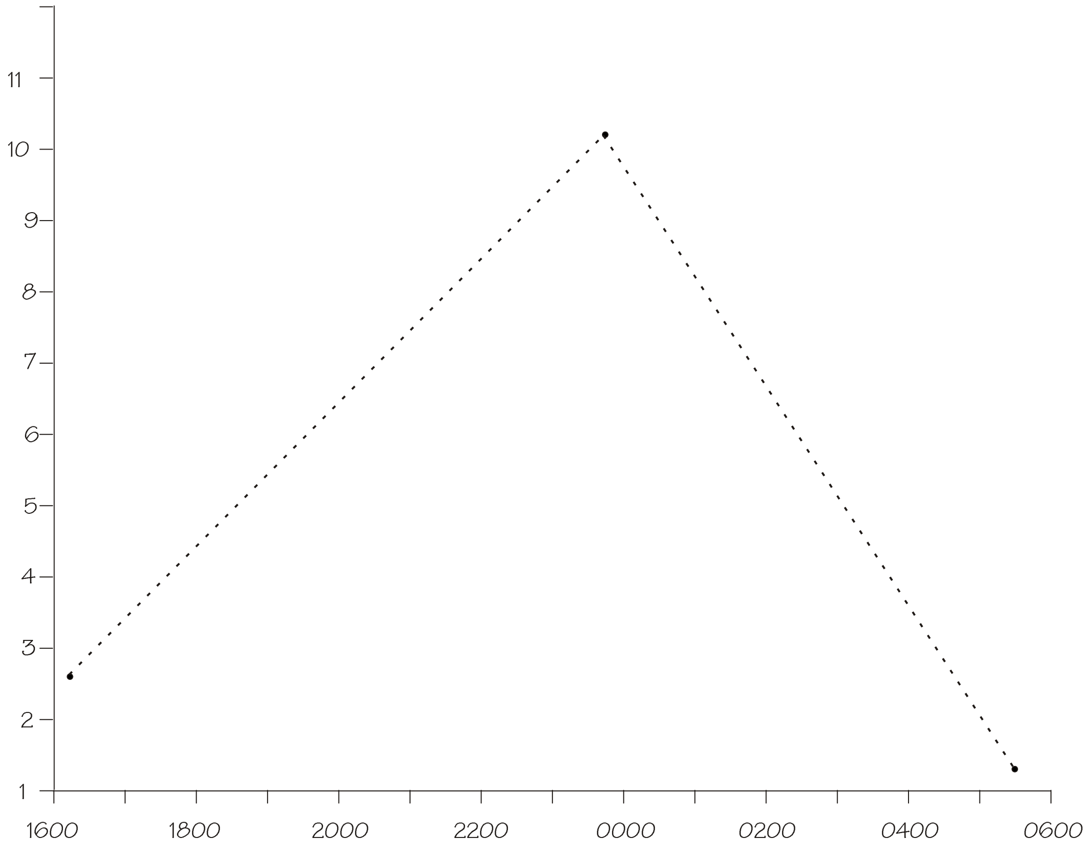
Step 2: Build a graph that is of a proper scale to capture these heights and times.

For instance, assume there was a low tide of 2.6 feet at 1615, and it is 1800 as you are cruising into your anchorage. You want to find a spot to anchor, and veer enough rode that you will neither touch bottom during the night, nor drag anchor. High tide will be 10.2 feet at 2345, followed by a low tide of 1.3 feet at 0530.

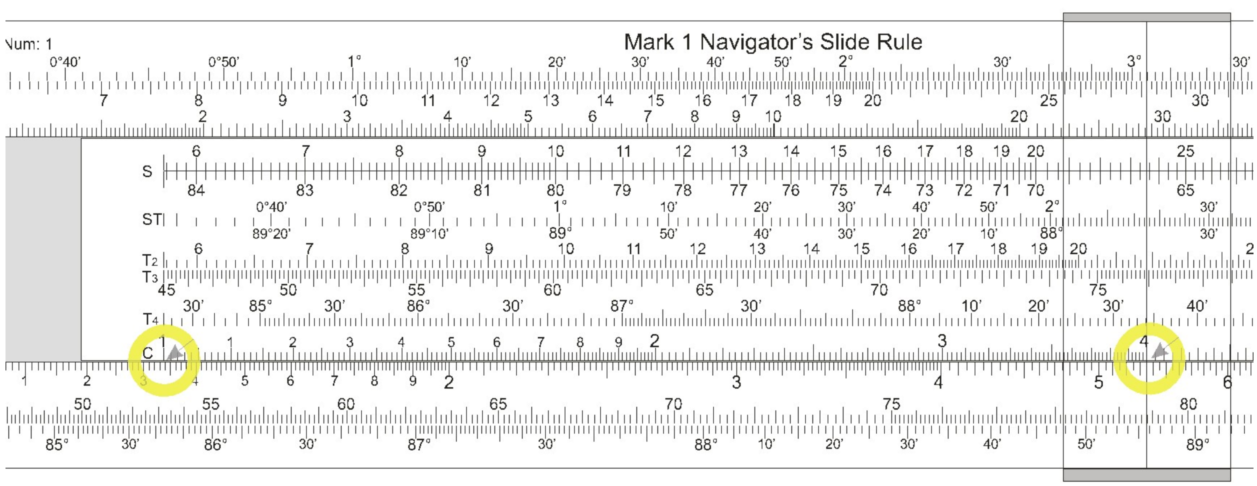
Here is what you basic graph looks like.

[63] When I lived in Japan, I owned a 13' dinghy that I stored on a hand-trailer. You can see a configuration similar to what I had at http://www.bobgoethe.com/images/cicada.jpg. The boat ramp I used was concrete, and had a drop off at the end of it. As long as the tide was above a certain level, I could launch and retrieve my boat easily. If it was below that level, it was impossible to haul the steel trailer up, together with 280 lb. of boat on it, over the end of the concrete ramp. In that case, I would have to wait for the tide to go to its minimum, then come back up before I could retrieve my boat.

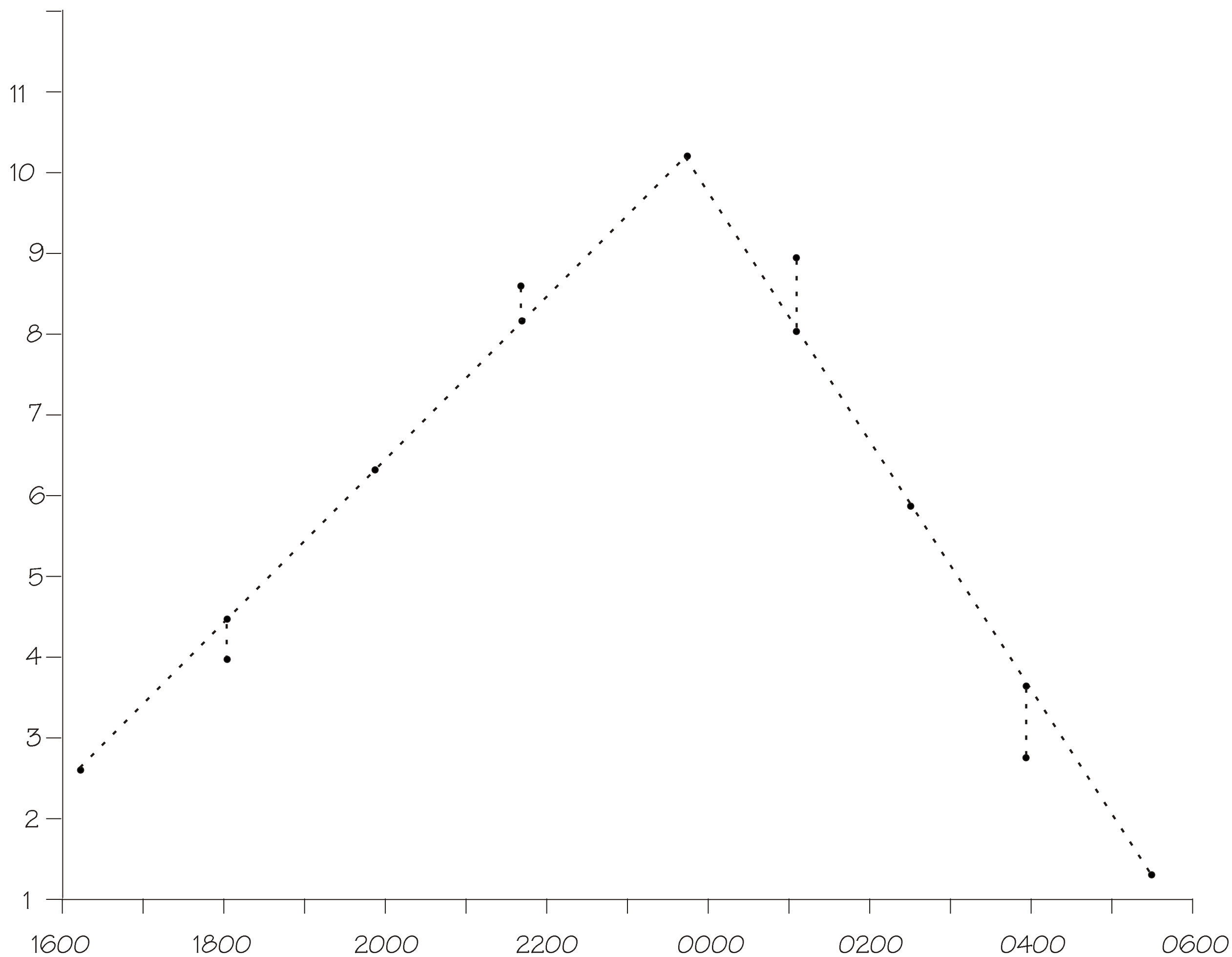
The local authority published tide tables in a little book that would fit in your hip pocket. I took to carrying a log book that was also small enough to fit in my hip pocket. Using the tide chart booklet itself as a straight edge, I became good enough at creating a graph of intermediate tide heights in my log book that I could predict the critical tide height for dinghy retrieval to within 5 or 10 minutes.



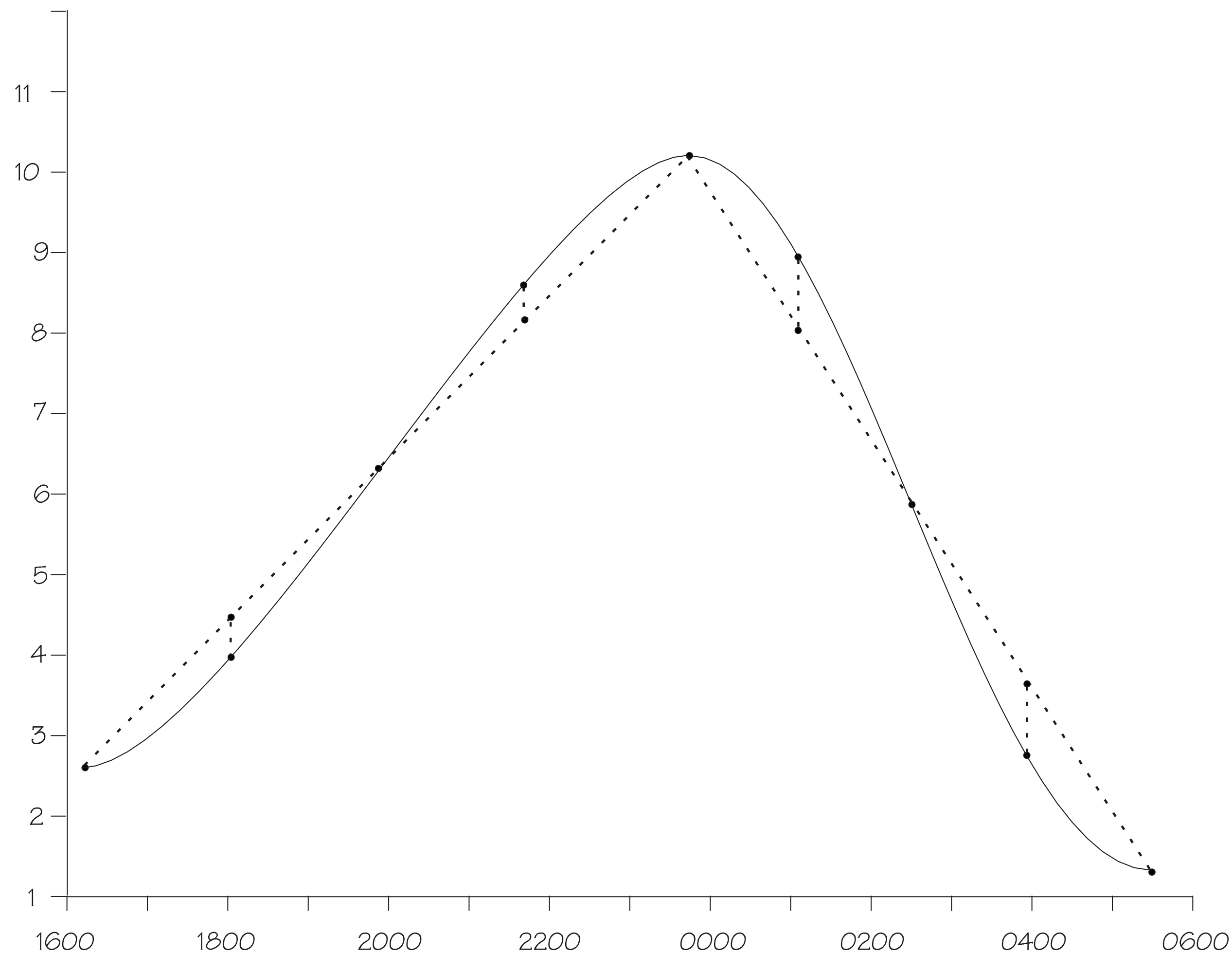
Step 3: Mark the lines at the half and quarter distances. Notice that the scale of inches on your Mark 1 is in tenths of an inch rather than quarters/eighths/etc. This will make it a bit easier for you to figure out the length of a quarter line. For instance, if you figure your line is 5.35" long, you can do division with your slide rule to sort out that ¼ of that is 1.34".



Step 4: Take 1/10th of the tidal range, and make a mark above/below your ¼ and ¾ marks. So for the tide from 1615 to 2345, the range is 7.6 feet. So make a mark at the equivalent of 0.76 feet on your graph. Do the same for the tide from 2345 to 0530, based on its range of 8.9 feet. Your graph now looks like this.



Step 5: Assuming your tide follows a sine curve (a pretty safe bet, most of the time, in a region with two high tides per day), you can draw a smooth curve connecting dots and get a pretty close picture of what your tide looks like.



From this graph, you can figure out what the tidal height is at the moment you drop anchor; what it will be a maximum tide tonight, and what the low tide will be in the morning. If you anchor in a place where you need at least 2 feet of tide above the chart datum to avoid touching bottom, you know that you will need to up-anchor by 0400, or 0415 at the latest.

§55. *Plotting Longitude*

The conventional way to determine longitude on a VP-OS Universal Plotting Sheet is to use a set of dividers together with the scale in the lower right corner of the page.[64]

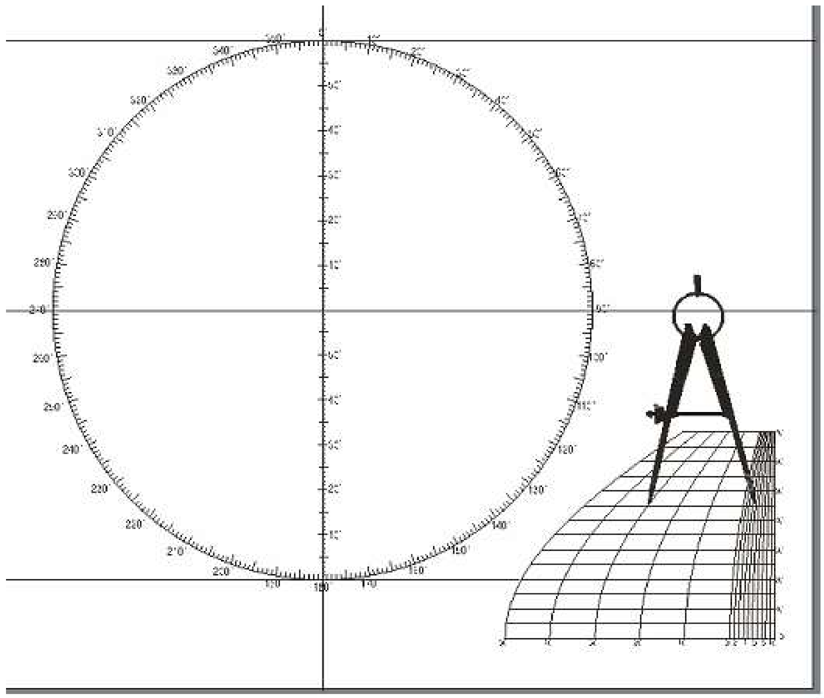


Figure 3.1

This is still a good method, and if you are comfortable with it, the by all means carry on. However, it is possible you may find it quicker to calculate longitudes using the scale of minutes on the Mark 1 Navigator's Slide Rule.

As you might imagine, there is a fixed, mathematical relationship between latitude and the width of a degree of longitude:

Width of degree of longitude in nautical miles = cos(latitude) × 60.

This can be calculated graphically by using the scale of minutes on the backside of the Mark 1 Navigator's Slide Rule. Rotate it so its offset from horizontal is equal to your latitude.

Construct a plotting sheet for North 20°, West 70°. Let's assume that your DR is North 20° 08', West 69° 23', and your AP is North 20° 00', West 69° 36'.

Center the scale on the Mark 1 at the center point of the plotting sheet, then rotate the rule by 20°. Make a mark next to the 23' point on the Mark 1, for your DR, and another at the 36' point, for your AP.

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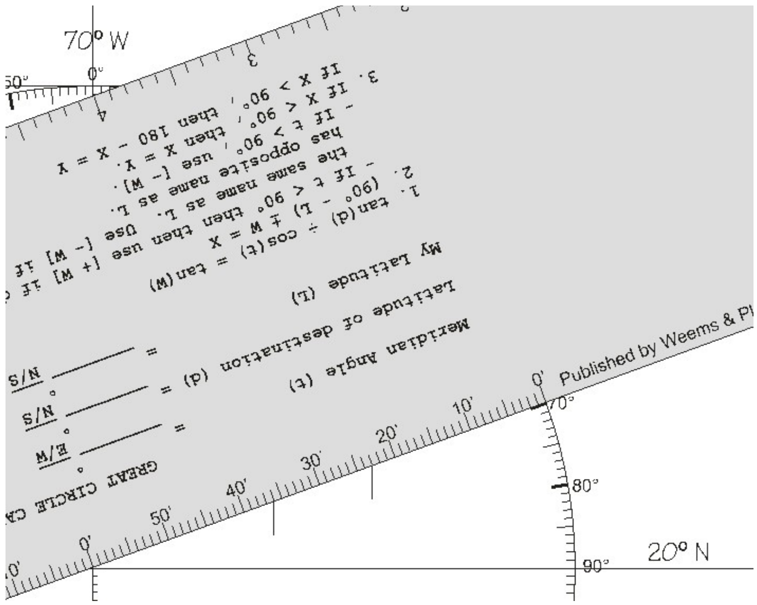
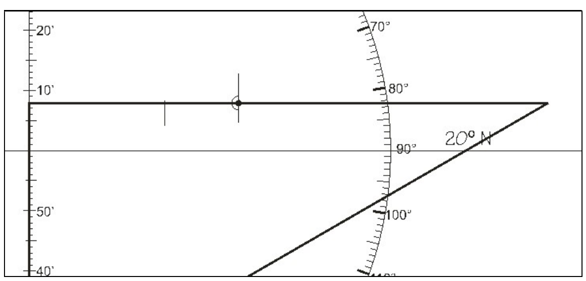
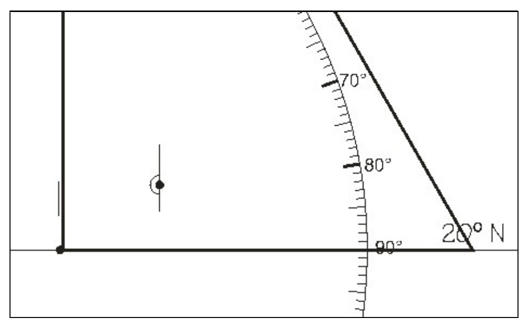
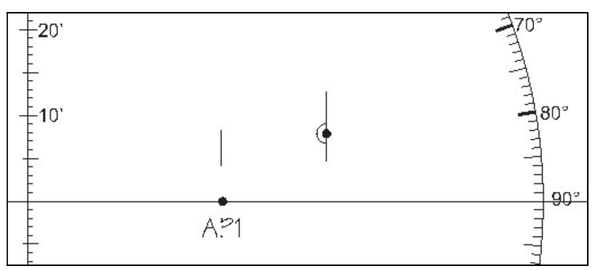
[64] This discussion assumes that you are using a VP-OS sheet published by Weems and Plath, or one that uses precisely the same scale.



Use your triangle to sort out your latitude, and position a mark for your DR. Use the triangle to extend your mark at 36' down to 20° for your AP.







Now, to fix this concept in your head, let's do another problem. Assume your AP is at North 53° 00', West 145° 49'. Rotate the ruler from horizontal by 53°.[65] Draw a line down from 49' on the Mark 1 rule.

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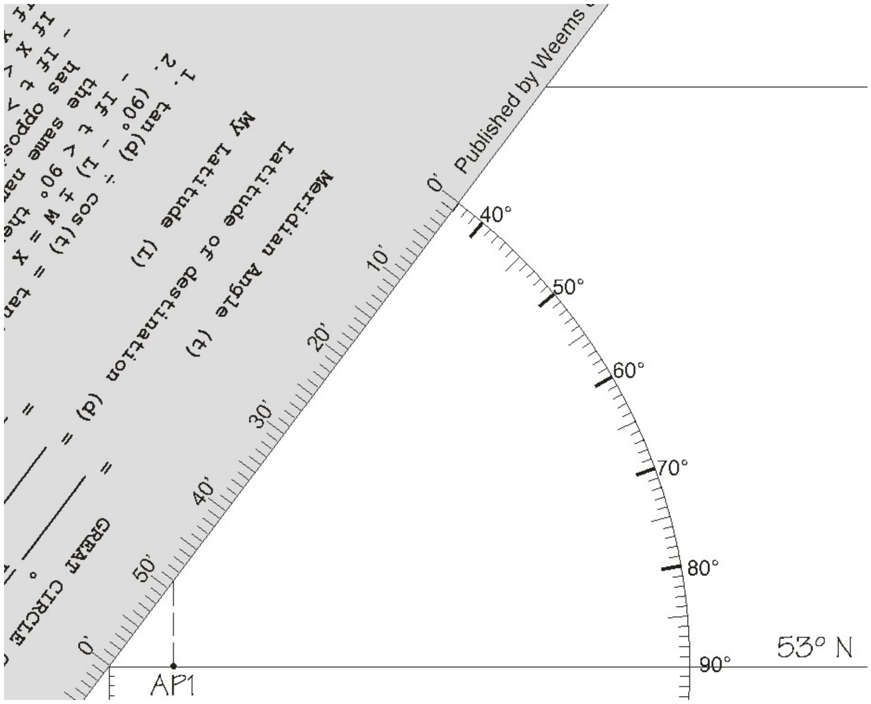
[65] Be careful to note this does not mean rotating the ruler until it lines up with 53° on the compass rose, but 53° away from horizontal.


Published by Weems
My latitude (L)
Latitude of destination (d)
Meridian Angle (t)
GREAT CIRCLE
40°
50°
60°
70°
80°
90°
53° N
AP1

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[66] Bowditch first published his handbook of navigation in 1802. It was updated and republished several times in his lifetime, and later by his son. In 1867, the copyright and plates were purchased by the Hydrographic Office of the US Navy. It has been published, through more than 75 editions, by the US government ever since.

[67] I am obliged to Bill Anderson of Crofton, British Columbia, for bringing this article to my attention.

[68] For anybody interested in traditional navigation, this is an enormously important resource. Do a search from there on "Bygrave" and you can review an extensive discussion – spanning many years – on different mathematical approaches to solving the navigational triangle. You can even access a statistical analysis of the accuracy of Bygrave's cosine/tangent equations compared to the more-conventional cosine/sine equations, as solved using a 10" straight slide rule. See https://tinyurl.com/y3fumm2r and https://tinyurl.com/y5e6kqpp.

Members of this group have used PVC tubing to construct their own cylindrical slide rules, similar to the original Bygrave, and there are hundreds of postings related to Amelia Earhart's 1937 navigation across the Pacific Ocean. Further, this is really the ONLY place where you can dialog with people who, after they have gone to see *The Martian* (2015) with Matt Damon, muse aloud about the technical requirements for performing celestial navigation on Mars.

It was this group's discussion of the Bygrave formulas, in fact, that helped to determine which equations would be used in this volume for the solution of great circles/sight reduction. This in turn shaped the design of the Mark 1 Navigator's Slide Rule, as it was optimized to solve these particular equations.